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Gevrey Well-Posedness of the Hyperbolic Prandtl Equations

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In honor of Professor Chao-Jiang Xu on the occasion of his 65th birthday

Abstract. We study 2D and 3D Prandtl equations of degenerate hyperbolic type, and establish without any structural assumption the Gevrey well-posedness with Gevrey index ≤ 2. Compared with the classical parabolic Prandtl equations, the loss of the derivatives, caused by the hyperbolic feature coupled with the degeneracy, cannot be overcame by virtue of the classical cancellation mechanism that developed for the parabolic counterpart. Inspired by the abstract Cauchy-Kowalewski theorem and by virtue of the hyperbolic feature, we give in this text a straightforward proof, basing on an elementary L^2 energy estimate. In particular our argument does not involve the cancellation mechanism used efficiently for the classical Prandtl equations.

AMS subject classifications: 35Q35, 35Q30, 76D10, 76D03

Key words: Hyperbolic Prandtl boundary layer, well-posedness, Gervey space, abstract Cauchy-Kowalewski theorem.

1 Introduction and main result

The classical Prandtl equation was derived by Prandtl in 1904 as a model to describe the behavior of the flow near the fluid boundary. It is a degenerate

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parabolic type equation, losing one order tangential derivatives due to the absence of tangential diffusion. The mathematical study on the classical Prandtl boundary layer has a long history with various approaches developed and so far it is well-explored in various function spaces, see, e.g., [3, 7, 9–12, 14–16, 19, 20, 22, 25, 26, 29, 41–43] and the references therein. Among those works, there are two main settings. One is referred to the Sobolev well/ill-posedness that usually requires Oleinik's monotonicity assumption (cf. [3, 31, 33]), and another is referred to the Gevrey (including analyticity) well-posedness without any structural assumption. Note that Oleinik's monotonicity condition is crucial for the Sobolev well-posedness theory. On one hand, the classical solution was established by Oleinik (see for instance [33]) and two groups Alexandre *et al.* [3] and Masmoudi and Wong [31] independently proved the existence and uniqueness theory in the Sobolev spaces by using energy method. On the other hand, without Oleinik's monotonicity, the Prandtl equation may be ill-posed in Sobolev space (Gevrey space more precisely) that was observed by Gérard-Varet and Dormy [11]. Inspired by the ill-posedness result in [11, 29], it is natural to expect the Gevery well-posedness whenever the initial data are in Gevery class ≤ 2 without any structural assumption. This has been confirmed recently by Dietert and Gérard-Varet [9] and Li *et al.* [18] for the 2D and the 3D Prandtl equations, respectively, after the earlier works of [7,12,22] in Gevrey class and Sammartino and Caflisch's work [39] in the analytic setting. Recently, similar problems have been explored for MHD boundary layer equations that are of Prandtl type equations with the strong background magnetic fields; for instance the stabilizing effect of the magnetic fields has been justified recently by Liu *et al.* [28] (see also [21,24] for the further development), and the ill-posed in Sobolev space (more precisely, in Gevrey spaces with index >2) is established by [24,27] and the Gevrey well-posedness is proven by [23]. The aforementioned works are mainly concerned with the local-in-time existence. We refer to Xin and Zhang's work [41] on global weak solutions under the monotonicity condition and the recent works of Paicu and Zhang [35] and Wang *et al.* [40] on global small analytic and Gevrey solutions, respectively. The global analytic solutions to MHD boundary layer system were obtained recently by Liu and Zhang [30] and Li and Xie [17].

This work aims to study the well-posedness theory of the hyperbolic version of Prandtl equations. The hyperbolic boundary layer equations can be derived similarly as the classical Prandtl system, just following the Prandtl's ansatz when analyzing the asymptotic expansion with respect to the viscosity of the hyperbolic Navier-Stokes equations which are complemented with the no-slip boundary conditions. The hyperbolic version of Navier-Stokes equation by adding a small hyperbolic perturbation to the classical one, was initiated by Cattaneo [6].