CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.SO-2022-0001

Vol. **3**, No. 4, pp. 810-840
December 2022

Weyl Almost Periodic Solutions in Distribution of Clifford-Valued Stochastic High-Order Hopfield Neural Networks with Delays on Time Scales

Yongkun Li* and Xiaoli Huang

Department of Mathematics, Yunnan University, Kunming, Yunnan 650091, P.R. China

Received 6 January 2021; Accepted 24 April 2022

Abstract. In this paper, we first propose a concept of Weyl almost periodic random processes on time scales, including the concepts of Weyl almost periodic random processes in p-th mean and Weyl almost periodic random processes in distribution. Then, using the Banach fixed point theorem, time scale calculus theory and inequality techniques, the existence and stability of Weyl almost periodic solutions for Clifford-valued stochastic high-order Hopfield neural networks with time-varying delays on time scales are studied. Even when the system we consider degenerates into a real-valued system, our results are new. Finally, a numerical example is given to illustrate the feasibility of our theoretical results.

AMS subject classifications: 34N05, 34K50, 34K14, 34K20, 92B20

Key words: Weyl almost periodic random processes on time scales, Clifford-valued neural network, Weyl almost periodic solution in distribution, global exponential stability, time scales.

---

1 Introduction

Weyl almost periodicity is a generalization of Bohr almost periodicity and Stepanov almost periodicity [1, 2]. The spaces composed of Bohr almost periodic functions and Stepanov almost periodic functions are Banach spaces. Therefore, there have been many research results on Bohr almost periodic solutions and Stepanov almost periodic solutions of differential equations. However, the space composed of Weyl almost periodic functions is not a Banach space, which makes it difficult to study the existence of Weyl almost periodic solutions of differential equations. Up to now, there are few results on the existence of Weyl almost periodic solutions of differential equations. In particular, there are almost no results on the existence of Weyl almost periodic sequence solutions of

*Corresponding author. *Email addresses:* `yklie@ynu.edu.cn` (Y. Li), `xlhuang@mail.ynu.edu.cn` (X. Huang)

difference equations. The same is true for stochastic differential equations and stochastic difference equations. We know that stochastic differential equations have been developed for more than 80 years and have been widely used in the fields of economy, biology, physics, automation and so on. At the same time, in a certain sense, stochastic differential equations can be regarded as a generalization of deterministic differential equations. Therefore, the study of Weyl almost periodic solutions of stochastic differential equations has important theoretical and practical significance.

Although continuous time systems and discrete-time systems are equally important in theoretical analysis and practical application, discrete-time systems are more convenient for computer processing. However, it is not necessary to study continuous time systems and discrete-time systems respectively, because the study of dynamic equations on time scales can unify the study of the corresponding problems of continuous time systems and discrete-time systems [3–5]. Therefore, it is of great significance to study the dynamic equations on time scales.

On the one hand, higher-order Hopfield neural networks, which are described by differential equations, have attracted the attention of many researchers because they perform better than lower-order Hopfield neural networks in convergence speed, storage capacity and fault tolerance. Also, because the dynamics of neural networks plays an important role in their design, implementation and application, and time delay is inevitable for real systems [6, 7], a lot of research on the dynamics of high-order Hopfield neural networks with delays have been done and many results have been achieved [8–15].

On the other hand, in the real world, neural network systems are usually inevitably affected by external uncertain disturbances that may be regarded as random. Therefore, neural networks with random disturbances have attracted more and more attention. In addition, almost periodic oscillations are important dynamics of neural networks. In the past few decades, the existence of almost periodic solutions of neural networks has been widely studied. Although there have been many studies on Bohr almost periodic solutions and Stepanov almost periodic solutions of neural networks [16–23], there are very few results on Weyl almost periodic solutions of neural networks [24], and there are fewer results on Weyl almost periodic solutions of stochastic neural networks. Therefore, it is an important and challenging work to study the existence of Weyl almost periodic solutions of stochastic neural networks.

In addition, although real-valued, complex-valued and quaternion-valued neural networks have their own very important applications, Clifford-valued neural networks include real-valued, complex-valued and quaternion-valued neural networks as their special cases, and have more advantages than real-valued neural networks in dealing with high-dimensional data and geometric transformation [25–28]. Therefore, in recent years, they have aroused the interest of more and more researchers [29–35]. However, because the multiplication of Clifford algebra does not meet the commutative law, there are few research results on the dynamics of Clifford-valued neural networks, so it is necessary to further study it.

Inspired by the above discussion, the main purpose of this paper is (i) to unify the