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Lattice Boltzmann Modeling of Cholesteric Liquid Crystal Droplets Under an Oscillatory Electric Field

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Abstract. We numerically study the dynamics of quasi-two dimensional cholesteric liquid crystal droplets in the presence of a time-dependent electric field, rotating at constant angular velocity. A surfactant sitting at the droplet interface is also introduced to prevent droplet coalescence. The dynamics is modeled following a hybrid numerical approach, where a standard lattice Boltzmann technique solves the Navier-Stokes equation and a finite difference scheme integrates the evolution equations of liquid crystal and surfactant. Our results show that, once the field is turned on, the liquid crystal rotates coherently triggering a concurrent orbital motion of both droplets around each other, an effect due to the momentum transfer to the surrounding fluid. In addition the topological defects, resulting from the conflict orientation of the liquid crystal within the drops, exhibit a chaotic-like motion in cholesterics with a high pitch, in contrast with a regular one occurring along circular trajectories observed in nematics drops. Such behavior is found to depend on magnitude and frequency of the applied field as well as on the anchoring of the liquid crystal at the droplet interface. These findings are quantitatively evaluated by measuring the angular velocity of fluid and drops for various frequencies of the applied field.

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1 Introduction

Nematic liquid crystals are an example of soft material in which the local alignment of anisotropic-shaped molecules they are made of is described by a unit magnitude director field $\mathbf{n}$ with head-tail symmetry. Cholesteric liquid crystals, on the contrary, are chiral systems in which the locally favoured state of the director field is a twist deformation in the direction perpendicular to the molecules [1–3]. Such helical arrangement is characterized by a helix pitch p_0, a quantity measuring the distance over which the director rotates by 2π.

Of particular relevance to us are cholesteric liquid crystal droplets, highly confined chiral soft fluids that have found vast application in several sectors of modern industry, ranging from photonics [4,5] and laser beams [6] to microlasers [7], optics [8], displays [9] and, more recently, as active material [10–12]. In these objects the order of the director is crucially affected by the anchoring of the liquid crystal at the droplet interface [13–19]. Indeed, under confinement, the typical helical structure of the cholesteric may conflict with that imposed at the boundaries, often favouring the formation of topological defects (or disclinations) whose nature can decisively condition mechanical and optical properties of the liquid crystal [15, 20, 21]. While, over the years, considerable efforts have been addressed to theoretically investigate the physics of cholesteric droplets and their associated defect structure at equilibrium [15, 16, 22–28], only recently a number of numerical works have been dedicated to pinpointing their response under an external driving, such as a heat flux [29] or an electric field [30, 31]. Such works have been inspired by experiments showing for example that, if subject to a temperature gradient, cholesteric drops are set into rotation due to either a thermomechanical torque mechanism [32–34] or to Marangoni flows [29,35]. A rotation can be alternatively triggered by applying a uniform (i.e. time independent) and large enough electric field, giving rise to a torque applied to the liquid crystal confined within the drops [36–39]. Further experiments have also shown that angular velocity and shape of such rotating drops can be controlled by tuning an oscillatory electric field coupled to the liquid crystal subject to a thermal gradient [40].

In a previous work [30] we numerically studied the response of a quasi-two dimensional cholesteric drop dispersed in an isotropic fluid solely subject to an electric field coupled to the liquid crystal, and we showed that its dynamics and that of the defects critically depend on magnitude and direction of the field as well as on elasticity and pitch of the liquid crystal. If the field is non uniform, such as a rotating one with constant frequency, the defects display a persistent periodic motion occurring with an angular speed generally lower than that set by the field, due to the anisotropy of the liquid crystal. In this work we go one step further and consider a couple of cholesteric drops in an isotropic fluid subject to a time-dependent electric field $\mathbf{E}(\mathbf{r},t)$ rotating at constant frequency ω. Droplet coalescence is prevented by including a surfactant accumulated at their interfaces. The theoretical framework used to describe the droplet physics relies on well-established continuum prescriptions [41], in which a small number of continuum fields, such as concentration and ordering of the liquid crystal, amount of surfactant, den-