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High-Order Spectral Difference Gas-Kinetic Schemes for Euler and Navier-Stokes Equations

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Dedicated to Professor Tao Tang on occasion of his 60th birthday.

Abstract. High-order spectral difference gas-kinetic schemes (SDGKS) are developed for inviscid and viscous flows on unstructured quadrilateral meshes. Rather than the traditional Riemann solver, the spectral difference method is coupled with the gas-kinetic solver, which provides a time-accurate flux function at the cell interface. With the time derivative of the flux function, a two-stage fourth-order time-stepping method is adopted to achieve high-order accuracy with fewer middle stages. The stability analysis for the linear advection equation shows that fourth-order spatial and temporal discretization SDGKS is stable under CFL condition. Quantitatively, the fourth-order SDGKS is around 8% more efficient than the traditional one with the Riemann solver and the strong stability preserving five-stage fourth-order Runge-Kutta method. Both steady and unsteady tests obtained by SDGKS compare well with analytic solutions and reference results.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

Key words: Spectral difference method, high-order method, gas-kinetic scheme, two-stage fourth-order time stepping, Navier-Stokes equation.

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1. Introduction

The high-order gas-kinetic schemes (GKS) under the finite volume framework have been developed in the last decade [10]. The GKS flux is based on a time evolution solution of the Bhatnagar-Gross-Krook (BGK) model [3]. Compared with traditional Riemann solver, the highlights of GKS include these:

i) The gas distribution function at interfaces contains the evolution from the upwind flux vector splitting to the central difference Lax-Wendroff type discretization.

ii) The inviscid and viscous fluxes are evaluated simultaneously.

iii) The GKS flux has multi-dimensional properties [34], where both normal and tangential derivatives of flow variables are involved in the time evolution of gas distribution function.

iv) The time-accurate gas evolution updates the solution at the cell interface which can be used in the construction of high-order compact schemes [38].

v) The multi-stage multi-derivative (MSMD) methods can be applied in GKS, and higher-order time accuracy with few middle stages can be achieved.

vi) The multi-scale unified GKS (UGKS) is also developed for the whole flow regime [13, 14].

The family of high-order GKS [8], based on the same WENO reconstruction, has favorable performance in efficiency, accuracy, and robustness, in comparison with the traditional high-order schemes with Riemann solver and Runge-Kutta (RK) time-stepping techniques. Owing to the multi-dimensional property in GKS flux, it captures flow structures, such as shear instabilities, much better than the schemes using the Riemann solver. Among those high-order GKS, the two-stage fourth-order method (S2O4) [21] seems to be the optimal choice and is efficient, accurate, and as robust as the second-order one. Besides, it has been applied to multicomponent flow [19], the direct simulation of compressible homogeneous turbulent flow [20], and hypersonic multi-temperature flow [4]. The high-order GKS has been successfully extended to the discontinuous Galerkin (DG) [16, 25, 26] and the correction procedure via reconstruction (CPR) [36] as well. And it has been applied within the finite difference framework on uniform grids [35]. In this paper, the high-order GKS will be developed on the unstructured quadrilateral meshes under the spectral difference framework for the first time.

The spectral difference (SD) method was firstly proposed in [15, 32] for simplex elements and has been studied in the past decades. It combines the advantages of finite-volume and finite-difference methods, such as geometric flexibility and high computational efficiency. The three-dimensional SD method has also been developed on hexahedral meshes by Sun *et al.* [29] and was used to simulate turbulent channel flow in [11, 23, 24]. However, the original form of the SD method is known to have instability on triangular elements, losing its popularity on simplex elements. Later, Balan *et al.* [1] proposed a stable