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Rough Heston Models with Variable Vol-of-Vol and Option Pricing

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Dedicated to the memory of Professor Zhongci Shi

Abstract. In this paper, a rough Heston model with variable volatility of volatility (vol-of-vol) is derived by modifying the generalized nonlinear Hawkes process and extending the scaling techniques. Then the nonlinear fractional Riccati equation for the characteristic function of the asset log-price is derived. The existence, uniqueness and regularity of the solution to the nonlinear fractional Riccati equation are proved and the equation is solved by the Adams methods. Finally the Fourier-cosine methods are combined with the Adams methods to price the options.

AMS subject classifications: 60G22, 60G55, 65R20, 91G20

Key words: Rough Heston model, option pricing, Hawkes process, fractional differential equations, Fourier-cosine methods.

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1 Introduction

The classical Heston model invented by Heston [27] has been received large popularity in the financial market due to that it generates very reasonable shapes of the implied volatility surface and there is analytic formula for the characteristic function of the asset log-price. In a recent paper by Gatheral, Jaisson and Rosenbaum [24], it is empirically shown that the financial time series of the realized volatility exhibit much rougher than that of the Brownian motion for the volatility in the classical Heston model. The dynamics of the log-volatility behave essentially as a fractional Brownian motion with Hurst parameter H less than 1/2. Therefore, many researchers have focused their studies on fractional Brownian motion and use it to express the squared volatility process in the financial models. The fractional Brownian motion can be expressed by the Mandelbrot-Van Ness representation (see e.g., [40]) which contains an integral process $\int_0^t (t-s)^{H-1/2} dW_s$. Since the integral process has Hölder regularity $H-\epsilon$ for any $\epsilon>0$, in the rough Heston model, the kernel of the form $(t-s)^{H-1/2}$ is often introduced in the squared volatility process. For example, in [17] it gives a rough Heston model on a filtered probability space $(\Omega,\mathcal{F},\mathbb{F},\mathbb{P})$ of the form

$$dS_t = S_t\sqrt{V_t}dW_t, \tag{1.1a}$$

$$V_t = V_0 + \frac{\gamma}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}(\theta - V_s)ds + \frac{\gamma}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\nu\sqrt{V_s}dB_s, \tag{1.1b}$$

where the parameters γ, θ, ν, V_0, S_0 are positive constants, $\alpha := H+1/2 \in (1/2,1)$, and W and B are two Brownian motions with correlation coefficient $\rho \in [-1,1]$. If $\alpha = 1$, then the rough Heston model (1.1a)-(1.1b) coincides with the classical Heston model. Therefore, the rough volatility model (1.1a)-(1.1b) can be viewed as the rough extension of the classical Heston model. However, the rough squared volatility model (1.1b) is not Markovian and the variance process is not anymore a semi-martingale. Therefore, the use of the Monte-Carlo methods for the option pricing under the rough Heston model becomes quite intricate (see [9]) and the analytic method in [27] is also hard to adapt to the rough Heston model.

Jaisson and Rosenbaum [31] investigate the nearly unstable Hawkes process and show that it asymptotically behaves like an integrated Cox-Ingersoll-Ross model (see [11]), i.e., the squared volatility process of the classical Heston model. The Hawkes process is a kind of self-exciting and mutually exciting point processes introduced by [28,29]. The essential property of Hawkes process is that the occurrence of any event increases the probability of further events occurring. Over that last decades, the probabilistic and statistical analysis of Hawkes processes has known several interesting developments, driven by the growing use of Hawkes processes in