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Dynamic Analysis of Stochastic Spruce Budworm Differential Model with Time Delay*

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Abstract In this paper, we consider a stochastic spruce budworm differential model with time delay. Based on the nonnegative initial conditions, the existence and uniqueness of the global positive solution are easily found. Then, we obtain the ultimate boundedness of solution in mean under the same conditions. Furthermore, we verify that the sample Lyapunov exponent of solution is less than a positive constant. Finally, numerical examples are presented to show the consistency of the theoretical results.

Keywords Spruce budworm model, Stochastic perturbation, Global solution.

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1. Introduction

The spruce budworm, found in spruce-fir forests in the United States and eastern Canada, is one of the most destructive insect species. On the basis of [3], its periodic outbreaks may result in the loss of large amounts of food and natural resources. According to records from the United States and Canada in [12], spruce budworm outbreaks have occurred approximately every 40 years since the 18th century, each lasting about 10 years and causing enormous damage to forest resources. In [2, 7], Canadian scholars Ludwing et al., and Dwyer et al., established the following classical spruce budworm model

$$\mathrm{d}u(t) = \left[ru(t)(1 - \frac{u(t)}{K}) - \frac{Bu^2(t)}{A + u^2(t)} \right] \mathrm{d}t, \tag{1.1}$$

where $u(t)$ represents the density of spruce budworm population at time t, $r > 0$ represents the population growth rate, and K represents the environmental carrying capacity. $B > 0$ is the predation rate of predators or parasites of u, and $A > 0$ means the saturate effect of the predators or parasites at the high density of u. In [17], Wang and Yeh investigated the bifurcation of model (1.1) with reaction diffusion.

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The spruce budworm generally completes its life-cycle within one year in [13], and can be divided into four stages: egg, larva, cocoon and adult, among which the larva can be divided into six juvenile stages. From an egg to the second juvenile stage, it accounts for about three quarters of the entire reproductive cycle. During this period, there is almost no activity of aphids, and birds mainly prey on the budworm after this period of time. In 2008, Vaidya and Wu in [15] treated the egg to the second instar larvae as immature stage and a time-delay differential equation with age structure, for the adult spruce budworm was established. The spruce budworm model with time delay can be described as follows

$$\mathrm{d}u(t) = [-ru(t) + \beta e^{-c\tau} u(t-\tau) e^{-\alpha u(t-\tau)} - \frac{Bu^2(t)}{A^2 + u^2(t)}]\mathrm{d}t \tag{1.2}$$

with initial conditions

$$u(s) = \rho(s) \ for \ s \in [-\tau, 0], \quad \rho \in C([-\tau, 0], R_+). \tag{1.3}$$

Here, $R_+ = [0, +\infty)$, $u(t)$ represents the density of adult spruce budworm, $r > 0$ is the average mortality of adult budworm, $B > 0$ is the predation rate of birds, and $A > 0$ indicates the population density of spruce budworm when the predation rate reaches a half of the maximum. $\tau > 0$ is the time taken from birth to maturity. $c > 0$ is the average mortality of spruce budworm larvae. $b(u) = \beta u e^{-\alpha u}$ represents the birth function of spruce budworm and $\beta, \alpha > 0$.

Nevertheless, in the natural world, the spruce budworm model is inevitably more or less influenced by environment noises. In [10], May proposed that parameters in the system like environmental capacity and population growth rate had exhibited random fluctuations due to environmental noises. Many scholars have studied population behavior with random disturbances. For example, Peng and Zhang studied the stochastic predator-prey model with non-constant mortality rate in [11]. In [6], Liu and Zhu studied the stability of a budworm growth model with stochastic perturbation. Moreover, Song et al., [14] explored the dynamical behavior of stochastic Beddington-DeAngelis predator-prey model with distributed delay. Therefore, in [1,16], the delayed differential equations of stochastic spruce budworm are more suitable to model the data of (1.2). We assume that the average mortality r is disturbed with $r \to r - \sigma \mathrm{d}B(t)$, $B(t)$ is one-dimensional Brownian motion with $B(0) = 0$ defined on a complete probability space $(\Omega, \{\mathcal{F}_t\}_{t \geq 0}, P)$, and σ^2 is the intensity of the noise. In addition, consider $c = 0$, so that the mortality of spruce budworm larvae is 0. Then, we get the following stochastic function

$$\mathrm{d}u(t) = \left[-ru(t) + \beta u(t-\tau) e^{-\alpha u(t-\tau)} - \frac{Bu^2(t)}{A^2 + u^2(t)}\right] \mathrm{d}t + \sigma u(t) \mathrm{d}B(t). \tag{1.4}$$

This article focuses on the following aspects. In Section 2, we provide preliminaries results. In Section 3, some properties of (1.4), such as the existence and uniqueness global positive solution of (1.4) with initial values (1.3), ultimate boundedness and the sample Lyapunov exponent of (1.4), are given. Examples and numerical simulations are carried out to support the results in Section 4. Finally, we briefly summarize the work of this paper.