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Dynamical Property Analysis of a Delayed Diffusive Predator-prey Model with Fear Effect*

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Abstract In this paper, we study a delayed diffusive predator-prey model with fear effect and Holling II functional response. The stability of the positive equilibrium is investigated. We find that time delay can destabilize the stable equilibrium and induce Hopf bifurcation. Diffusion may lead to Turing instability and inhomogeneous periodic solutions. Through the theory of center manifold and normal form, some detailed formulas for determining the property of Hopf bifurcation are presented. Some numerical simulations are also provided.

Keywords Delay, Diffusion, Predator-prey, Turing instability, Hopf bifurcation.

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1. Introduction

In recent years, reaction-diffusion models have been widely and profoundly applied in biomathematics. Many scholars have paid attention to them and studied their dynamics [13,14,16,25–28]. It was found that changes in population density depend not only on time, but also on space. The predator and prey are non-homogeneous in space. Thus, diffusion is a phenomenon that cannot be ignored. In [26], a cross-diffusive predator-prey model with pack predation-herd behavior was considered. Yang, Zhang and Yuan primarily investigated the Turing pattern caused by cross-diffusion. In [16], Peng, Li and Zhang studied a toxin producing phytoplankton-zooplankton system with prey-taxis. They mainly discussed prey-taxis induced Turing instability and the local existence of the nonconstant positive steady state. These papers show that diffusion may lead to Turing pattern and spatial inhomogeneous periodic oscillation, which are worth investigating. Motivated by them, we also introduce diffusion terms into our model.

The purpose of this article is to investigate the stability of the positive equilibrium, Turing instability and Hopf bifurcation of the new system. This paper is organized as follows. In Section 2, we give a detailed description about the formation of our model. In Section 3, we analyze the stability of the positive equilibrium, Turing instability and the existence of Hopf bifurcation. In Section 4, we study the

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property of Hopf bifurcation. In Section 5, some numerical simulations are shown. Finally, a conclusion is presented in Section 6.

2. Model formulation

In this section, we will introduce the process of model formulation. Let us lay the foundation for investigating the global dynamics of the system.

In the current study, a logistic equation is often used to model the growth of the prey population in the absence of a predator. Suppose that the functional response of predator-prey interaction is Holling type II. Then, the population densities of prey and predator at the time T are denoted as X and Y respectively. First, consider a two-dimensional Rosenzweig-MacArthur [18] predator-prey model with the form

$$\begin{cases} \dot{X} = R_0 X(1 - \frac{X}{K_0}) - \frac{CAXY}{B+X}, \\ \dot{Y} = \frac{AXY}{B+X} - DY. \end{cases} \tag{2.1}$$

All parameters are positive and their biological meanings are shown in Table 1.

Table 1. Biological description of parameters in the paper

Parameter	Definition	Parameter	Definition
X	Prey density	Y	Predator density
R_0	Prey intrinsic growth rate	K_0	Prey carrying capacity
A	Maximum predation rate of predator	B	Half-saturation coefficient of predator
C^{-1}	Conversion efficiency of predator	D	Natural mortality rate of predator
E	Density restriction of predator	K	Fear parameter

With the development of the biomathematics, various predator-prey models have been studied in [2, 8, 9, 11, 23, 24]. They found that the fear effect plays a crucial role in the ecosystem. The physiological feature or behaviors of the prey population may change due to the fear of predators, including the alteration of foraging [1], breeding [4], inhabiting [17] and so on, which further affect their growth rate. Then, consider the modified growth rate of the prey $\frac{R_0}{1+KY}$. Moreover, the prey needs some time to evaluate the predation risk for the perception of the dangers, and then it makes the above changes. Hence, the fear effect does not reduce the growth of the prey population instantaneously, but it needs time delay. Based on (2.1), Panday et al. [15] proposed the following model and studied the permanence, local and global stability, as well as Hopf bifurcation of this delayed differential equation

$$\begin{cases} \dot{X} = \frac{R_0}{1+KY(T-T_1)} X(1 - \frac{X}{K_0}) - \frac{CAXY}{B+X}, \\ \dot{Y} = \frac{AXY}{B+X} - DY. \end{cases} \tag{2.2}$$

As a matter of fact, many factors may affect the dynamics of a system such as time delay, diffusion terms, density restriction and competition [3,6,7,10,20,21]. It is well-known that when predator population becomes too large, a density restriction may exist due to the intraspecific competition denoted as E. After introducing