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An Accelerated Algorithm Involving Quasi-ϕ-Nonexpansive Operators for Solving Split Problems

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Abstract In this paper, an algorithm of inertial type for approximating solutions of split equality fixed point problems involving quasi-ϕ-nonexpansive maps is proposed and studied in the setting of certain real Banach spaces. Weak and strong convergence theorems are proved under some conditions. Some applications of the theorems are presented. The results presented extend and improve some existing results. Finally, some numerical illustrations are presented to support our theorems and their applications.

Keywords Fixed point, Quasi-ϕ-nonexpansive, Inertia.

MSC(2010) 47H05, 47H09, 47H10, 47H20.

1. Introduction

Let E_1, E_2 and E_3 be real Hilbert spaces, and let D and Q be nonempty closed and convex subsets of E_1 and E_2 respectively. Let $S : E_1 \to E_3$, $T : E_2 \to E_3$ be bounded linear mappings, and let $B : E_1 \to E_1$ and $A : E_2 \to E_2$ be nonlinear mappings such that $F(B)$ and $F(A)$ are nonempty respectively. The split equality fixed point problem (SEFPP) is to find

$$u \in F(B) \quad \text{and} \quad v \in F(A) \quad \text{such that} \quad Su = Tv. \tag{1.1}$$

The problem was first introduced by Moudafi [29], and since then, it has been studied by many researchers (see, e.g. [14, 33, 36, 37] and the references therein). It allows asymmetric relations between the two variables u and v, and also covers many problems such as decomposition methods for partial differential equations (PDEs), and has applications in game theory and in intensity modulated radiation therapy (see, e.g. [9]).

Remark 1.1. If $E_2 = E_3$ and $T = I$, the SEFPP (1.1) reduces to the split common fixed point problem, which was first studied by Censor and Segal [10]. The problem is to find $u \in E_1$ with

$$u \in F(B) \quad \text{and} \quad Su \in F(A).$$

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Various algorithms for approximating solutions of the SEFPP (1.1) have been introduced and studied by numerous researchers in Hilbert spaces and Banach spaces more general than Hilbert spaces (see, for example, [1,9,12,13,21,24] and the references therein).

In 2019, Chidume, Romanus and Nyaba [22] considered the following algorithm in the setting of some Banach spaces:

Algorithm 1.

$$\begin{cases} x_1 \in X_1, y_1 \in X_2, z_n \in J_{X_3}(Sx_n - Ty_n); \\ x_{n+1} = J_{X_1}^{-1}(a_n J_{X_1} u_n + (1-a_n) J_{X_1} B u_n),\ u_n = J_{X_1}^{-1}(J_{X_1} x_n - \gamma S^* z_n); \\ y_{n+1} = J_{X_2}^{-1}(a_n J_{X_2} v_n + (1-a_n) J_{X_2} A v_n),\ v_n = J_{X_2}^{-1}(J_{X_2} y_n + \gamma T^* z_n), \end{cases} \tag{1.2}$$

where X_1 and X_2 are Banach spaces that are uniformly smooth and 2-uniformly convex with weak continuous duality maps J_{X_1} and J_{X_2}, respectively, X_3 is a Banach space with duality map J_{X_3}, A and B are quasi-ϕ-nonexpansive mappings, T and S are bounded linear mappings, $\{a_n\}$ is a sequence in $(0,1)$ and γ is a constant that satisfies a certain condition. They proved that the sequence generated by Algorithm 1 converges weakly to a solution of the SEFPP (1.1).

Many efforts have been devoted to improving the convergence speed of the existing iterative algorithms (see, e.g. [2,8,15,16,18,26]). An inertial algorithm was introduced by Polyak [31] to accelerate the process of solving the convex minimization problem. Since then, various iterative algorithms involving inertial extrapolation term have been proposed by numerous authors (see [6–8, 11, 19, 23, 28, 31]).

Motivated by the research on inertial acceleration technique, in this paper, we incorporate the inertial extrapolation term in Algorithm 1 of Chidume, Romanus and Nyaba [22] for approximating solution(s) of the SEFPP to get an algorithm which accelerates approximation of solution of the SEFPP in some Banach spaces. Unlike in the theorem Chidume, Romanus and Nyaba [22] where weak convergence was established under weak sequencial continuity of the duality mappings, we prove weak convergence of theorem in the setting of Opial spaces. In addition, we prove strong convergence under semi-compactness condition on the quasi-ϕ-nonexpansive maps. Furthermore, we give applications of our theorem to *split equality equilibrium problem, split equality variational inclusion problem* and *split equality problem.* Finally, some numerical examples are given to support our theorems.

2. Preliminaries

Let X be a real Banach space which is smooth and let $\phi : X \times X \to \mathbb{R}$ be a map given by

$$\phi(r,s) = \|r\|^2 - 2\langle r, Js\rangle + \|s\|^2, \quad \forall\, r, s \in X, \tag{2.1}$$

with J being the normalized duality map whose definition and properties on some Banach spaces can be found in, for example, [4]. Alber [4] first introduced this function, and since then numerous researchers have been studying it (see, for example, [3,17,20,27]). By the definition of ϕ, we can see that if X is a real Hilbert space, (2.1) reduces to $\phi(r,s) = \|r-s\|^2$, $\forall r, s \in X$. Furthermore, given $r, s, t, u \in X$, ϕ has the following properties

$$(\|r\| - \|s\|)^2 \le \phi(r,s) \le (\|r\| + \|s\|)^2,$$