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Dynamics Behavior of a Stochastic Predator-Prey Model with Stage Structure for Predator and Lévy Jumps*

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Abstract In order to study the effects of external environmental noise on the interaction dynamics between predator and prey populations, in this paper, we develop a predator-prey model with the stage structure for predator and Lévy noise. By constructing an appropriate Lyapunov function, we first prove that the proposed model exists the uniqueness of global positive solution. Then, we analyze the persistence and extinction of the proposed model. Finally, we perform some numerical simulations to verify the correctness of the theoretical results.

Keywords Predator-prey model, stage structure, Lévy jumps, persistence and extinction

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1. Introduction

Predation relationships are the most common in nature, the research on predator-prey models has attracted the attention of many researchers [1–6]. These researches laid the foundation for the future work. For the typical predator-prey model, it is generally assumed that predators are equally capable of hunting prey species. But the physiology of species in nature is complex. In many species, individuals are only able to hunt when they are adults, and the immature predators have to rely on mature ones for nourishment. Thus predatory ability could be ignored, such as sparrows, penguins and so on. Recently, some scholars have paid attention to the predator-prey models with the stage structure, and they have done some work in this research direction [7–10]. In addition, scholars have developed many predator-prey models with different functional response functions, especially for the Holling type II functional response, which is the most commonly used and takes the form of $f(x) = \frac{bx}{1+mx}$, where b is the search rate and m is the search rate multiplied by the handling time [11–14].

Some research work on the predator-prey model with Holling type II functional response has been developed and investigated. For example, Wang and Chen [15] proposed and analyzed the following predator-prey model with Holling type II func-

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tional response and the stage structure for predator:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} =x(t)(r-ax(t))-\dfrac{bx(t)z(t)}{1+mx(t)}, \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} =\dfrac{kbx(t)z(t)}{1+mx}-(D+d_1)y(t), \\ \dfrac{\mathrm{d}z(t)}{\mathrm{d}t} =Dy(t)-d_2z(t), \\ x(0) =x_0, y(0)=y_0, z(0)=z_0, \end{cases} \tag{1.1}$$

where $x=x(t)$, $y=y(t)$ and $z=z(t)$ denote the densities of prey, immature and mature predators at time t, respectively. r is the intrinsic growth rate of the prey, a represents the intraspecific competition rate of the prey, b is the consumption rate of mature predators to prey, $k(0<k<1)$ is the conversion efficiency of prey into newborn immature predators, d_1 and d_2 represent the death rates of immature and mature predators, D is the rate at which immature predators become mature predators, $x(0)=x_0$, $y(0)=y_0$ and $z(0)=z_0$ are initial values. All the parameters are positive constants. By defining the basic reproduction number of the predator $R_0=\frac{kbDr}{d_2(a+mr)(D+d_1)}$ as the average number of offsprings produced by a mature predator in its lifetime, Georgescu and Morosanu [16] showed that if $R_0\leq 1$, then the prey-only equilibrium $\left(\frac{r}{a},0,0\right)$ is globally asymptotically stable on $\mathbb{R}^3_+$, while if $R_0>1$, the prey-only equilibrium $\left(\frac{r}{a},0,0\right)$ is unstable, and there exists only one positive equilibrium.

However, there exists certain limitation for the deterministic model (1.1), and it cannot reflect the effect of environmental factors on the dynamical behavior of model (1.1). Thus, by taking into account the influence of external environment noise, Liu [17] introduced the standard white noise into model (1.1) and then obtained the following stochastic model:

$$\begin{cases} \mathrm{d}x=\left[x(r-ax)-\dfrac{bxz}{1+mx}\right]\mathrm{d}t+\sigma_1x\mathrm{d}B_1(t), \\ \mathrm{d}y=\left[\dfrac{kbxz}{1+mx}-(D+d_1)\,y\right]\mathrm{d}t+\sigma_2y\mathrm{d}B_2(t), \\ \mathrm{d}z=[Dy-d_2z]\,\mathrm{d}t+\sigma_3z\mathrm{d}B_3(t), \end{cases} \tag{1.2}$$

where σ_1^2, σ_2^2, σ_3^2 are the intensities of the environment white noise, $B_1(t)$, $B_2(t)$ and $B_3(t)$ are mutually independent standard Brownian motions with $B_1(0)=B_2(0)=B_3(0)=0$.

In addition, sudden environmental disturbance, such as hurricanes, earthquakes, floods, etc, can also have a significant impact on the predator and prey species. In order to better understand the effects of these phenomena on the dynamics of the predator-prey model, it is worth studying the predator-prey model with jumps process. Applebaum and Siakalli [18] extended Mao's techniques to the case of nonlinear stochastic differential equations driven by Lévy jumps and studied the probability stability, almost certainty stability and moment exponential stability of the stochastic differential equation. Zhao and Yuan [19] pointed out that Lévy noise can affect the optimal harvesting strategy of inshore and offshore fisheries. Liu and Bao et al. [20, 21] analyzed the Lotka-Volterra system affected by Lévy noise, and the results indicated that Lévy noise has a certain effect on the dynamics of the