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A New Fixed-Time Dynamical System for Absolute Value Equations

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Abstract. A novel dynamical model with fixed-time convergence is presented to solve the system of absolute value equations (AVEs). Under a mild condition, it is proved that the solution of the proposed dynamical system converges to the solution of the AVEs. Moreover, in contrast to the existing inversion-free dynamical system (C. Chen *et al.*, Appl. Numer. Math. 168 (2021), 170–181), a conservative settling-time of the proposed method is given. Numerical simulations illustrate the effectiveness of the new method.

AMS subject classifications: 90C33, 65K05

Key words: Absolute value equation, fixed-time convergence, dynamical system, numerical simulation.

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1. Introduction

To solve the system of absolute value equations (AVEs) is to find an $x \in \mathbb{R}^n$ such that

$$Ax - |x| - b = 0, \tag{1.1}$$

where $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$ and $|x|$ represents the componentwise absolute value of the

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unknown vector x. The AVEs (1.1) is a special case of the generalized absolute value equations (GAVEs)

$$Cx - D|x| - c = 0, \tag{1.2}$$

in which $C, D \in \mathbb{R}^{m\times n}$, $x \in \mathbb{R}^n$ and $c \in \mathbb{R}^m$. The GAVEs (1.2) is originally introduced by Rohn in [23] and further investigated in [6, 14, 21] and the references therein. The AVEs (1.1) and the GAVEs (1.2) are closely related to many mathematical programming problems, such as the linear complementarity problem (LCP) [8,14,16,21], the horizontal LCP (HLCP) [19], the generalized LCP (GLCP) [16] and others, see e.g. [15,16,21]. In addition, they have relevance to the system of linear interval equations [22].

Solving the GAVEs (1.2) is generally NP-hard [14]. Moreover, when the GAVEs (1.2) is solvable, it concludes from [21] that checking whether the GAVEs (1.2) has a unique solution or multiple solutions is NP-complete. Throughout this paper, we will assume that A is invertible and $\|A^{-1}\| < 1$, and thus the AVEs (1.1) has a unique solution for any $b \in \mathbb{R}^n$ [16]. The interested reader is referred to [7,19,24,27,28] for more discussions about the unique solvability of the AVEs (1.1).

There are many ways to solve AVEs (1.1), as long as it is uniquely solvable. The class that we are interested in is the continuous solution schemes. We will briefly present some of the existing work presented below. To this end, we recall two reformulations of the AVEs (1.1). The first reformulation is obtained in the case that 1 is not the eigenvalue of A, and, specifically, the AVEs (1.1) can be reformulated as an LCP [16]: Find a $u \in \mathbb{R}^n$ such that

$$u \geq 0, \quad (A+I)(A-I)^{-1}u + q \geq 0, \quad \big\langle u, (A+I)(A-I)^{-1}u + q\big\rangle = 0 \tag{1.3}$$

with

$$q = \big[(A+I)(A-I)^{-1} - I\big]b, \quad u = (A-I)x - b. \tag{1.4}$$

Obviously, if u^* is a solution of the LCP (1.3), then $x^* = (A-I)^{-1}(u^* + b)$ is a solution of the AVEs (1.1). The second reformulation is that the AVEs (1.1) is equivalent to the following GLCP [16]: Find an $x \in \mathbb{R}^n$ such that

$$Q(x) = Ax + x - b \geq 0, \quad F(x) = Ax - x - b \geq 0, \quad \big\langle Q(x), F(x)\big\rangle = 0. \tag{1.5}$$

By utilizing the reformulation (1.3) of the AVEs (1.1), some dynamical systems are constructed to solve the AVEs (1.1). For instance, the following dynamical model:

$$\begin{aligned} &\text{state equation:} && \frac{\mathrm{d}u}{\mathrm{d}t} = P_\Omega\big[u - \lambda h(u,\beta)\big] - u, \\ &\text{output equation:} && x = (A-I)^{-1}(u+b) \end{aligned}$$

is used by Mansoori *et al.* [18] to solve the AVEs (1.1), where

$$\begin{aligned} h(u,\beta) &= e(u,\beta) - \beta M e(u,\beta), \\ e(u,\beta) &= u - P_\Omega\big[u - \beta(Mu+q)\big], \end{aligned}$$