Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.OA-2022-0189

Vol. **16**, No. 3, pp. 752-768
August 2023

Geometric Design of Letters in Times Roman Font via RBF Meshless Collocation Method

Lionel Amuzu, C. S. Chen, Kwesi Acheampong and Huiqing Zhu*

School of Mathematics and Natural Sciences, University of Southern Mississippi, Hattiesburg, MS, USA

Received 1 December 2022; Accepted (in revised version) 7 March 2023

Abstract. Two mathematical models in the context of boundary value problems are proposed for the geometric design of letters in Times Roman font. We adopt radial basis function meshless collocation method for numerically solving the two proposed mathematical models in 2D and 3D. In this paper, Bézier curves play an important role in the design of the letters. Three examples with simply and multiply-connected domains in 2D and 3D are presented to demonstrate the visual effect of the letters in Times Roman font.

AMS subject classifications: 65N35, 65H10

Key words: Kansa method, Bézier curve, implicit surface, computer graphics, image reconstruction, meshless method, radial basis functions.

1. Introduction

In the literature of computer graphics and computer-aided design, various mathematical models and numerical techniques have been developed for the reconstruction of curves, and surfaces [1, 3, 7, 11–14]. There is no unique curve or surface for a given set of points cloud data to represent these data points. The Implicit model is one of the popular approaches used to create an implicit function representing the given points cloud data. Mathematical modeling via partial differential equations (PDEs) has been applied to obtain the implicit function and thus create curves/surfaces to represent the given points cloud data visually [7, 11, 12, 14]. An implicit curve/surface is a collection of data points $f(\mathbf{x}), \mathbf{x}\in\mathbb{R}^d, d=2,3$, such that $f(\mathbf{x})=0$, where $f(\mathbf{x})$ is an implicit function. Such a curve/surface is called the zero set of f, which can be used to identify the locus of all points $\mathbf{x}$ on a specific curve/surface by simply checking the sign of $f(\mathbf{x})$. Note that the mathematical model that produces the implicit function is capable of identifying whether a given data point is located inside, outside, or on the curve/surface by check-

*Corresponding author. *Email address:* Huiqing.Zhu@usm.edu (H. Zhu)

ing the sign of the implicit function $f(\mathbf{x})$. Further, each value of the implicit function $f(\mathbf{x})$ represents a different level of curve/surface. Such a procedure for reconstructing the curve/surface is also called the level set method [10]. The implicit function can be taken as the solution of a PDE model [7, 11, 12, 14]. After solving the PDE, we produce an implicit function $f(\mathbf{x})$ for the construction (or design) of closed and connected curves/surfaces from a given data set of points using the level set method. For the implicit function $f(\mathbf{x}) = \gamma$, with γ a constant, each γ produces a level curve/surface. We can choose a proper γ to construct or design the desired curve/surface.

The current paper is motivated by [12] in which a potential field model was proposed in the design of letters in the Times Roman font. The method of fundamental solutions (MFS) was adopted as a numerical tool for solving the given boundary value problem. The main advantage of the MFS is its simplicity. However, since the fundamental solution is singular, a fictitious boundary outside the domain is required in the numerical implementation. The selection of the fictitious boundary could be problematic for complicated geometries. In this paper we propose an alternative PDE model, the modified Helmholtz equation with a Dirichlet boundary condition. For the proposed PDE model, a numerical method needs to be selected for its solution. In recent years, the radial basis functions collocation method (RBFCM), or the so-called Kansa method [5], has become a simple and effective meshless method for solving a large class of PDEs. Note that the simplicity for solving the proposed PDE model is a major consideration for adopting the method in question. Other RBF collocation methods can also be employed to achieve this purpose [4, 6, 8]. Since radial basis functions (RBFs) are highly smooth functions, they are ideal for constructing and/or designing curves and surfaces. The novelty of the current paper is the proposed mathematical model for the geometric modeling for the curve/surface design in 2D and 3D. Furthermore, with the proposed model, we have further improve the visual quality of the designed letters in Times Roman font without the need of a fictitious boundary as shown in [12].

One of the main challenges in RBF application is the determination of a good shape parameter to achieve high accuracy. Various techniques have been proposed for this purpose. However, the optimal shape parameter value is often problem-dependent. Despite intensive research, this issue is still considered an outstanding research topic. In our proposed PDE model, the goal is to recover the geometry of the boundary and is quite different from the traditional way of finding the solution inside the domain. As such, no matter how we choose the shape parameter, the solution inside the domain could be different, but the solution on the boundary should remain the same. Hence, the determination of the shape parameter is not an important issue in our solution process.

The paper is organized as follows. In Section 2, we propose two PDE models for the geometric design of letters in the Times Roman font. Section 3 introduces the Kansa method as a numerical tool for solving the boundary value problems proposed in Section 2. In Section 4, we present three examples to design and construct four letters in Times Roman font. Some conclusions and ideas for future work are outlined in Section 5.