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Edge Detectors Based on Pauta Criterion with Application to Hybrid Compact-WENO Finite Difference Scheme

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Abstract. In the last two decades, many edge detection methods have been developed and widely used in image processing for edge detection and the hybrid compact-WENO finite difference (hybrid) schemes for solving the system of hyperbolic conservation laws with solutions containing both discontinuous and complex fine-scale structures. However, many edge detection methods include the problem-dependent parameters such as the high order multi-resolution (MR) analysis (Harten, JCP, 49 (1983)). Therefore, we combined the Tukey's boxplot method with MR analysis (Gao et al., JSC, 73 (2017)) to overcome this problem in a sense. But the Tukey's boxplot method needs to sort the data at the beginning of Runge-Kutta time integration method, which is relatively time-consuming and inefficient. In this study, we employ the PauTa criterion and remove the problem-dependent parameters in the MR analysis. Furthermore, two new edge detection approaches, which are based on second-order central difference scheme and Ren's idea (Ren et al., JCP, 192 (2003)), are also proposed. The accuracy, efficiency and robustness of the hybrid scheme with the new edge detectors are verified by numerous classical one- and two-dimensional examples in the image processing and compressible Euler equations with discontinuous solutions.

AMS subject classifications: 65M10, 78A48

Key words: Edge detection, Pauta criterion, multi-resolution, hybrid.

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1 Introduction

The nonlinear system of hyperbolic conservation laws can be written compactly as

$$\frac{\partial \mathbf{Q}}{\partial t}+\nabla\cdot\mathbf{F}(\mathbf{Q})=0, \tag{1.1}$$

where $\mathbf{Q}$ and $\mathbf{F}(\mathbf{Q})$ represent the conservative variables and fluxes, respectively, together with appropriate initial and boundary conditions in a Cartesian domain. For example, the two-dimensional compressible Euler equation:

$$\frac{\partial \mathbf{Q}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}=0, \tag{1.2}$$

with

$$\mathbf{Q}=(\rho,\rho u,\rho v,E)^T,\quad \mathbf{F}=(\rho u,\rho u^2+P,\rho uv,(E+P)u)^T,\quad \mathbf{G}=(\rho v,\rho uv,\rho v^2+P,(E+P)v)^T,$$

where ρ is density, E is the total energy, $(u,v)^T$ is the velocity vector, $P=(\gamma-1)(E-\frac{1}{2}\rho(u^2+v^2))$ is the pressure and $\gamma=1.4$ is the specific heat ratio of ideal gas. For the nonlinear hyperbolic conservation laws, even if the initial conditions are sufficiently smooth, the solutions will be discontinuous [1, 14, 21, 27] over time. Therefore, the solutions of such nonlinear systems could create both complex fine smooth and large strong gradient flow structures dynamically in space and time.

The hybrid compact-WENO finite difference (hybrid) scheme, based on the high order nonlinear characteristic-wise weighted essentially non-oscillatory (WENO) scheme [1, 14, 27] and the high resolution spectral-like compact scheme, is widely used for capturing shocks and strong gradients accurately and resolving smooth scale structures of solutions of the system of hyperbolic conservation laws [4, 18, 20, 23, 26]. The key issue in any hybrid scheme is to design an accurate, robust, and efficient high order shock detection algorithm that is capable of determining the smoothness of the solution at any given grid point. Moreover, by assuming a discontinuity as an edge of an object in an image, the edge detection methods in image processing can be directly applied as a shock detector in the hybrid scheme. Therefore, in the last two decades, many edge detection methods have been developed for the edge detection and hybrid scheme for solving the system of hyperbolic conservation laws. Costa et al. [4, 5] presented the arbitrary order multi-resolution (MR) analysis of Harten [11] for recognizing non-smooth and smooth stencils. Don et al. [6] designed a conjugate Fourier shock detection algorithm using the conjugate Fourier partial sums and their derivatives to detect discontinuities. In literature [17], Li et al. discussed in detail the detection effect of various low-precision discontinuity detection methods, such as TVB method [3] and KXRCF method [15]. The results show that in the cases of complex structures, these low-order methods do not capture discontinuities accurately. Gao et al. [8] combined the Tukey's boxplot method [29] with the MR analysis to improve the robustness of the shock detection methods, which