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STABLE RECOVERY OF SPARSELY CORRUPTED SIGNALS THROUGH JUSTICE PURSUIT DE-NOISING*

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Abstract

This paper considers a corrupted compressed sensing problem and is devoted to recover signals that are approximately sparse in some general dictionary but corrupted by a combination of interference having a sparse representation in a second general dictionary and measurement noise. We provide new restricted isometry property (RIP) analysis to achieve stable recovery of sparsely corrupted signals through Justice Pursuit De-Noising (JPDN) with an additional parameter. Our main tool is to adapt a crucial sparse decomposition technique to the analysis of the Justice Pursuit method. The proposed RIP condition improves the existing representative results. Numerical simulations are provided to verify the reliability of the JPDN model.

Mathematics subject classification: 65F22, 68W25, 90C25.
Key words: Justice Pursuit De-Noising, Restricted isometry property, Corrupted compressed sensing, Signal recovery.

1. Introduction

The theory of compressed sensing (CS) has been a very active research field in recent years and has attracted much attention in signal processing, electrical engineering and statistics. CS is concerned with recovering high-dimensional sparse signals from a small number of linear measurements. Specifically, sparse recovery from fewer noiseless observations $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$ in standard CS is in general an ill-posed problem and can be solved by some optimization algorithms. A well-known effective algorithm is Basis Pursuit (BP)

$$\min \|\hat{\boldsymbol{x}}\|_1 \quad \text{s.t.} \quad \boldsymbol{y} = \boldsymbol{A}\hat{\boldsymbol{x}}, \tag{1.1}$$

where $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ is a sensing matrix with $m \ll n$, $\boldsymbol{y} \in \mathbb{R}^m$ is the measurement vector. In the case of bounded measurement noise, one observes $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}$, where $\boldsymbol{e} \in \mathbb{R}^m$ denotes a bounded noise vector with $\|\boldsymbol{e}\|_2 \leqslant \varepsilon$. A widely used approach is the following Basis Pursuit De-Noising (BPDN) method [9]:

$$\min \|\hat{\boldsymbol{x}}\|_1 \quad \text{s.t.} \quad \|\boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{x}}\|_2 \leqslant \varepsilon. \tag{1.2}$$

A central goal of CS is to recover the unknown sparse or nearly sparse signal $\boldsymbol{x} \in \mathbb{R}^n$ exactly or stably from the constrained method (1.1) or (1.2) based on $\boldsymbol{A}$ and $\boldsymbol{y}$. Here, a vector $\boldsymbol{x} \in \mathbb{R}^n$

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is called s-sparse if the number of nonzero elements in $\boldsymbol{x}$ is at most s and a vector $\boldsymbol{v} \in \mathbb{R}^n$ is said nearly s-sparse if the error of its best s-term approximation decays quickly in s [16]. Many recovery guarantees about the methods (1.1) and (1.2) have been well developed and readers can refer to [5, 8–12, 17, 34, 35, 42].

Different from classical CS, corrupted compressed sensing can deal with unbounded noises that appear in many settings, such as impulse noise, narrowband interference, malfunctioning hardware and transmission errors in the case where signals was sent over a noisy channel. In these cases, the measurement error may be sparse or approximately sparse with unbounded value. Its mathematical model can be represented as

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{f} + \boldsymbol{e} = [\boldsymbol{A}\ \boldsymbol{I}] \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{f} \end{bmatrix} + \boldsymbol{e}, \tag{1.3}$$

where the sparse corruption vector $\boldsymbol{f} \in \mathbb{R}^m$ may have extremely large elements and $\boldsymbol{I} \in \mathbb{R}^{m\times m}$ denotes the identity matrix. Several papers [7, 22, 26, 27, 30, 36, 38] considered the following constraint problem for the model (1.3):

$$\min_{\hat{\boldsymbol{x}}\in\mathbb{R}^n,\, \hat{\boldsymbol{f}}\in\mathbb{R}^m} \|\hat{\boldsymbol{x}}\|_1 + \lambda\|\hat{\boldsymbol{f}}\|_1 \quad \text{s.t.} \quad \|\boldsymbol{y} - (\boldsymbol{A}\hat{\boldsymbol{x}} + \hat{\boldsymbol{f}})\|_2 \leqslant \varepsilon, \tag{1.4}$$

where $\lambda > 0$ is a balance parameter. For the model (1.3), Lin *et al.* [27] proposed new algorithms for reconstructing signals that are nearly sparse in terms of a tight frame in the presence of bounded noise combined with sparse noise, and presented corresponding recovery guarantees. Li *et al.* [24] established sufficient conditions based on the restricted isometry property, which guarantee stable signal recovery from extended Dantzig selector and extended Lasso models. Foygel and Mackey in [15] used a convex programming method to recover $\boldsymbol{x}$ and $\boldsymbol{f}$ with or without prior information and provided new bounds for the Gaussian complexity of sparse signals, leading to a sharper recovery guarantee. Adcock *et al.* in [1] showed that the signal $\boldsymbol{x}$ and its corruption $\boldsymbol{f}$ can be recovered stably if the matrix $\boldsymbol{A}$ satisfies the generalized RIP of order $(2s_1, 2s_2)$ with

$$\delta_{2s_1,\,2s_2} < \frac{1}{\sqrt{1 + \left(\frac{1}{2\sqrt{2}} + \sqrt{\eta}\right)^2}}, \quad \eta = \frac{s_1 + \lambda^2 s_2}{\min\{s_1,\, \lambda^2 s_2\}},$$

(s_1 and s_2 are the sparsity of $\boldsymbol{x}$ and $\boldsymbol{f}$, respectively) and the generalized RIP will be defined below (see Definition 2.2).

Note that the measurement corruption may be sparse in some bases. Mathematically, we have

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{f} = \boldsymbol{\Phi} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{f} \end{bmatrix}, \tag{1.5}$$

where $\boldsymbol{A} \in \mathbb{R}^{m\times n}$, $\boldsymbol{B} \in \mathbb{R}^{m\times l}$, $\boldsymbol{\Phi} = [\boldsymbol{A}\ \boldsymbol{B}]$, $\boldsymbol{x} \in \mathbb{R}^n$ is a sparse unknown signal, $\boldsymbol{f} \in \mathbb{R}^l$ is a sparse corruption. For instance, when the measurements are corrupted by 60Hz hum, the corruption noise is sparse in the discrete Fourier basis. To reconstruct $\boldsymbol{x}$ (and $\boldsymbol{f}$) from the model (1.5), a popular Justice Pursuit (JP) method has been introduced by Laska *et al.* [23]

$$\min_{\hat{\boldsymbol{x}}\in\mathbb{R}^n,\, \hat{\boldsymbol{f}}\in\mathbb{R}^l} \|\hat{\boldsymbol{x}}\|_1 + \|\hat{\boldsymbol{f}}\|_1 \quad \text{s.t.} \quad \boldsymbol{y} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}\hat{\boldsymbol{f}}. \tag{1.6}$$