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A High Order Positivity-Preserving Discontinuous Galerkin Remapping Method Based on a Moving Mesh Solver for ALE Simulation of the Compressible Fluid Flow

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Abstract. The arbitrary Lagrangian-Eulerian (ALE) method is widely used in the field of compressible multi-material and multi-phase flow problems. In order to implement the indirect ALE approach for the simulation of compressible flow in the context of high order discontinuous Galerkin (DG) discretizations, we present a high order positivity-preserving DG remapping method based on a moving mesh solver in this paper. This remapping method is based on the ALE-DG method developed by Klingenberg et al. [17, 18] to solve the trivial equation $\frac{\partial u}{\partial t}=0$ on a moving mesh, which is the old mesh before remapping at $t=0$ and is the new mesh after remapping at $t=T$. An appropriate selection of the final pseudo-time T can always satisfy the relatively mild smoothness requirement (Lipschitz continuity) on the mesh movement velocity, which guarantees the high order accuracy of the remapping procedure. We use a multi-resolution weighted essentially non-oscillatory (WENO) limiter which can keep the essentially non-oscillatory property near strong discontinuities while maintaining high order accuracy in smooth regions. We further employ an effective linear scaling limiter to preserve the positivity of the relevant physical variables without sacrificing conservation and the original high order accuracy. Numerical experiments are provided to illustrate the high order accuracy, essentially non-oscillatory performance and positivity-preserving of our remapping algorithm. In addition, the performance of the ALE simulation based on the DG framework with our remapping algorithm is examined in one- and two-dimensional Euler equations.

AMS subject classifications: 65M60, 35Q31

Key words: Remapping, discontinuous Galerkin method, arbitrary Lagrangian-Eulerian, high order accuracy, multi-resolution WENO limiter, positivity-preserving.

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1 Introduction

For the numerical simulation of computational fluid dynamics (CFD), the Eulerian framework and the Lagrangian framework are two classical approaches. In the Eulerian framework, the fluid flows through a fixed mesh. It has strong robustness and can be used in the flow field with large deformation, but getting the precise physical interface is challenging. The Lagrangian framework, in which the mesh moves with the fluid velocity, can naturally and precisely track the interface between different materials and can capture the contact discontinuities sharply. Nevertheless, the computing process in the flow field with significant deformation may be terminated due to mesh distortion. The arbitrary Lagrangian-Eulerian (ALE) approach, which allows the grid points to move with an arbitrary velocity, could combine the best properties of the Lagrangian method and the Eulerian method. The ALE method has been favored in computing compressible flow with large deformation and is flexible in dealing with multi-material problems and the problems with moving domains. The simulations of the compressible Euler equations using the ALE technique have attracted a lot of scientific attention [2,13,14,17,21,38,39,43].

Generally, ALE methods can be implemented in two manners, i.e., the direct ALE method and the indirect ALE method. The indirect ALE method consists of three individual steps: a Lagrangian step, a rezoning step and a remapping step. In the Lagrangian step, the solution and the computational mesh are updated simultaneously. The nodes of the computational mesh are adjusted to more optimal positions during the rezoning step to improve the quality of the mesh and to relieve the error caused by mesh deformation. The remapping step is then performed, where the Lagrangian solutions are conservatively transferred from the old distorted Lagrangian mesh to the new rezoned mesh. The last two steps are as critical to the accuracy of the overall simulation as the first step since they must preserve the characteristic mesh features as well as the essential mathematical and physical properties of the Lagrangian solution.

In the application of computational fluid dynamics, we can classify the indirect ALE framework as based on the finite volume (FV) method [10, 21, 38] or the Runge-Kutta discontinuous Galerkin (RK-DG) method [13, 17, 39]. The numerical solution of the DG method is approximated by polynomials within each element. Hence it is easy to handle problems including discontinuities. It is also flexible for complex mesh geometries and unstructured meshes. Due to the excellent compactness and high order accuracy of the DG method, it has been widely applied to deal with fluid dynamic problems. The type of solution projected in the remapping phase is determined by the discretization methods applied in the Lagrange phase. The remapping stage of the indirect ALE-FV technique transfers the cell averages from the old mesh to the new mesh. There has been much research on this strategy given in [5,9,15,20–22,30]. Under the indirect ALE framework in conjunction with the DG approach, it is necessary to transfer the high order polynomials to a different set of high order polynomials defined on the new rezoned mesh while maintaining good performance. In this paper, we concentrate on the remapping step in the indirect ALE framework together with the DG method.