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Quenching Phenomenon of Solutions for Parabolic Equations with Singular Absorption on Graphs

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Abstract. In this paper, we study the quenching phenomenon of solutions for parabolic equations with singular absorption under the mixed boundary conditions on graphs. Firstly, we prove the local existence of solutions via Schauder fixed point theorem. Then, under certain conditions we obtain the estimates of quenching time and quenching rate. Finally, numerical experiments are shown to explain the theoretical results.

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Key Words: Graph; singular absorption; quenching; mixed boundary conditions.

1 Introduction

In this article, we mainly discuss the quenching phenomenon for the problem

$$\begin{cases} u_t(x,t)=\Delta_\omega u(x,t)+\beta u^{-q}(x,t)-\lambda u^{-p}(x,t), & (x,t)\in S\times(0,\infty), \\ B[u]=0, & (x,t)\in\partial S\times[0,\infty), \\ u(x,0)=u_0(x)>0, & x\in\bar{S}, \end{cases} \tag{1.1}$$

where $\beta>0$, $\lambda>0$, $p,q>0$, and u_0 satisfies the compatibility conditions. Δ_ω is the discrete Laplacian operator defined as

$$\Delta_\omega u(x,t)=\sum_{y\in\bar{S}}\omega(x,y)[u(y,t)-u(x,t)], \quad \forall x\in S,$$

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and weighted function $\omega(x,y):\bar{S}\times\bar{S}\to[0,+\infty)$ satisfies

$$\omega(x,x)=0,\ \forall x\in\bar{S},\quad \omega(x,y)=\omega(y,x),\quad \forall x,y\in\bar{S},\quad \omega(x,y)>0\Leftrightarrow x\sim y.$$

$B[u]=0$ represents the mixed boundary condition

$$\mu(x)\frac{\partial u}{\partial n}(x,t)-\sigma(x)u(x,t)=0.$$

Here $\mu(x)$ is a function greater than 0, and $\sigma(x)$ is nonnegative function defined on ∂S, and for all $x\in\partial S$ there is $\mu(x)-\sigma(x)>0$. $\partial u/\partial n$ denotes the discrete normal derivative as

$$\frac{\partial u}{\partial n}(x,t)=\sum_{y\in S}\omega(x,y)[u(x,t)-u(y,t)],\quad \forall x\in\partial S.$$

Eq. (1.1) models a polarization phenomenon in ionic conductor [1], and it can also be regarded as a limit case of chemical catalyst kinetic model or enzyme kinetic model [2-5]. Moreover, the Robin boundary condition means that the system has energy exchange with the outside medium, and it includes various boundary conditions.

Over the past few decades, the study of the internal structure of graphs has attracted the attention of many researchers in various fields [6,7]. In particular, the properties of discrete Laplace operator on graphs and the solutions of various boundary value problems have been studied by many authors because of its wide range of applications, ranging from solving diffusion equations on networks [8-10], to energy flow modeling through networks or molecular vibration[11,12]. In recent years, some scholars have begun to pay attention to the behavior study of the solution of evolution equations defined on the network structure. Many objects and their relationship are often represented by networks. In mathematics, the weighted graph is another name for a network. Vertices represent objects, while edges represent connections between objects. They are widely used to analyze discrete objects. Chung [11] first introduced some concepts on the graph, such as integral, directional derivative and gradient etc., and proved the uniqueness of global solutions of the inverse problem and the solvability of the first and second boundary value problems under appropriate monotonic conditions, which provide a theoretical basis for the partial differential equations on graphs.

In [13], Chung et al. studied the homogeneous Dirichlet boundary value problem for the ω-heat equation with absorption on a network

$$\begin{cases} u_t(x,t)=\Delta_\omega u(x,t)-u^q, & (x,t)\in G\times(0,+\infty),\\ u(x,t)=0, & x\in\partial G,t>0,\\ u(x,0)=u_0(x), & x\in G. \end{cases}$$

The absorption term denotes that the heat passing through networks is affected by the reactive forces proportional to the power of their potentials. In addition, the behavior of