INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING, *SERIES B*
Volume 4, Number 2, Pages 167–178

AN INTERPOLATION METHOD FOR RIGID CONTACT LENSES

GLEN DALE PEARSON JR., RAM IYER, AND STEVEN MATHEWS

Abstract. Custom designed rigid gas permeable lenses show promise for correcting the vision of subjects with significant higher order aberration beyond defocus (myopia and hyperopia) and astigmatism. Such aberrations could be coma, trefoil, spherical aberration, secondary astigmatism etc. Although, there exist several techniques for designing the central part of the front optic zone for subjects with significant higher order aberration, there does not exist a method for continuously differentiable interpolation of the corrected region and the rest of the front surface of the lens. This paper presents a method for C^1 interpolation based on the solution of the biharmonic equation with appropriate boundary conditions.

Key words. rigid contact lenses, higher order aberration, two dimensional cubic spline interpolation, biharmonic equation.

1. Introduction

In the early 1960's, it was Smirnov, who suggested that higher-order aberrations can be corrected with customized lenses to compensate for aberrations in individual eyes (page 19 in [1]). Aberrations in human vision, and optics, in general, may be characterized by a variety of methods such as the point spread function, the optical transfer function, Seidel polynomial expansion, or Zernike polynomial expansion [2, 3, 4]. Of these, the Zernike polynomial representation is especially useful in the design of contact lenses [2]. Traditionally, the majority of aberration correction methods dealt with a class called lower-order aberrations. This class consists of refractive errors such as Myopia (nearsightedness), Hyperopia (farsightedness), and Astigmatism. Glasses, contact lenses, and laser surgery can correct these types of refractive errors. Unfortunately, the eye can suffer from many more types of aberrations called higher-order aberrations, such as Coma, Trefoil, Spherical Aberration, and Secondary astigmatism, which are characterized by the coefficients in a Zernike expansion of the wavefront measured by an aberrometer. Such aberrations are especially prominent in patients suffering from a disease called keratoconus and significantly affects their vision [5].

In this paper, we present a method for C^1 interpolation of the corrected region on a customized rigid gas permeable contact lens with the rest of the front surface. There exist several techniques for the design of the front surface for the correction of aberrations [2, 6, 7, 8]. All of these methods will result in a modification of a circular region of the trial lens directly in front of the pupil. This leads to an edge dislocation at the boundary of the modified region, which leads to a C^1 interpolation problem. In this paper, we consider an aberration correction method by Guirao et. al., which accounts for decentrations and rotations of the lens. We then interpolate on the contact lens by solving the biharmonic equation with Dirichlet boundary conditions. This procedure completes the process of designing the front surface of the contact lens. We illustrate the process with data from a human subject.

Received by the editors March 10, 2013.
2000 *Mathematics Subject Classification.* 35R35, 49J40, 60G40.

2. Wavefront aberration correction using a contact lens

A wavefront is a constant phase solution to the scalar wave equation. The causal solution to the scalar inhomogenous wave equation is given by the Kirchhoff formula [4, 11]. The solution for the particular case of a point source is a spherical wave. Therefore, if a constant (in time) point light source is "placed" at the retina, the solution to wave equation yields a constant (in time) spherical wave solution within the vitreous humor of the eye. After passing through the lens in the eye, the opening in the iris, the anterior chamber, and the cornea, the solution in the space in front of the eye takes the form of a plane wave in the ideal case of no ocular aberrations (see figure 2.1 in [2] for an illustration). An imperfect optical system consisting of the cornea and lens will result in a wave that is not planar.

A device that "places" a point source at the retina and measures the reflected wave coming out of the cornea is a Shack-Hartmann wavefront sensor. The wavefront sensor shines a low-power laser through a patient's cornea and iris onto the retina. As the retina is a rough surface, the reflected light inside the eye is not a ray travelling back along the optical axis but a spherical wave.

The patient wears a lens of known front and back surface radii while being tested on a Sharck-Hartmann aberrometer. The back surface of the lens is designed by an optometrist after a corneal topography measurement. It is desirable for the patient to wear a trial lens during the aberrometry because the tear layer between the cornea and the contact lens is optically active, which is unaccounted for in the absence of the trial lens. Therefore, a lens designer only has to make corrections on the front surface of the trial lens itself to cancel the aberrations after an aberrometry measurement.

We present a simple lens design below based on the idea of equalizing optical path lengths so that the reflected wave coming out of the trial lens is a plane wave. Let $W(x, y)$ be the function of the wavefront reflected from the trial lens, and let $L(x, y)$ be the thickness of the contact lens. Furthermore, let L_{max} be the thickest part of the contact lens, and η_{glass} be the refractive index of the contact lens glass material. The expression

$$L(x, y) = L_{max} - \frac{W(x, y)}{\eta_{glass} - 1} \tag{1}$$

yields a simple method for designing a contact lens [2, 8]. Unfortunately, decentrations and rotations can occur each time the patient blinks (page 126 in [7]). As a result, it is essential that equation (1) take decentrations and rotations into account. Guirao, Cox, and Williams presented a method for optimizing the correction of the aberrations of the eye with decentrations and rotations (pages 126-127 in [7]). The method partially corrects each of the Zernike coefficients such that the average variance of the residual wavefront aberration for each decentration can be minimized. The interpolation method presented in the next section does not depend on the aberration correction method used. Any method including Equation (1) or the method in [7] may be used followed by an application of the interpolation method described in this paper.

2.1. Wavefront representation using Zernike polynomials. In 1934, Fritz Zernike introduced the concept of Zernike Polynomials. Unlike Seidel Aberrations, Zernike Polynomials represents aberrations of any order on the unit disk. Since