INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING, *SERIES B*
Volume 2, Number 2-3, Pages 231–247

SENSITIVITY ANALYSIS OF THE EARLY EXERCISE BOUNDARY FOR AMERICAN STYLE OF ASIAN OPTIONS

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(Communicated by Lubin G. Vulkov)

Abstract. In this paper we analyze American style of floating strike Asian call options belonging to the class of financial derivatives whose payoff diagram depends not only on the underlying asset price but also on the path average of underlying asset prices over some predetermined time interval. The mathematical model for the option price leads to a free boundary problem for a parabolic partial differential equation. Applying fixed domain transformation and transformation of variables we develop an efficient numerical algorithm based on a solution to a non-local parabolic partial differential equation for the transformed variable representing the synthesized portfolio. For various types of averaging methods we investigate the dependence of the early exercise boundary on model parameters.

Key words. Option pricing, American-style Asian options, early exercise boundary, fixed domain transformation

1. Introduction

Asian path dependent options belong to the class of financial derivatives whose payoff diagram depends not only on the underlying asset price but also on the path average of underlying asset prices over some predetermined time interval. Such path dependent options can be often found at commodities markets such as oil, grain trade, etc. At expiration, the payoff diagram of such options is less sensitive with respect to sudden changes of the underlying asset value. Therefore a holder of an Asian option can effectively hedge the risk arising from a sudden price jump close to expiry. Typically, the payoff diagram of an Asian path dependent option depends on either arithmetic or geometric average of the spot price of the underlying asset. Such contingent claims can be used as a financial instrument for hedging highly volatile assets or goods. We refer the reader to references [20, 9, 21, 8, 5, 4, 23, 15, 13, 22, 16, 10] discussing qualitative and quantitative aspects of pricing Asian path dependent options.

In this paper, we focus on a special subclass of Asian options. Namely, we will investigate the so-called average strike Asian call options. At the time of expiry $t = T$, a holder of such an option contract has the right (but not obligation) to purchase the underlying asset for the strike price given as the path average of underlying asset prices. This means that the terminal payoff diagram for such an option has the form: $V(S, A, T) = \max\{S - A, 0\}$, where $S = S_T$ is the spot price of the underlying asset, $A = A_T$ is the path average of the asset prices $S_t, t \in [0, T]$, over the time interval $[0, T]$ and $T > 0$ is the time of maturity.

Concerning the method how the path averaged asset price $A = A_t$ is calculated at a time $t \in [0, T]$, we can distinguish the following methods of averaging of the path $S_u, u \in [0, t]$:

Received by the editors January 15, 2011.
2000 *Mathematics Subject Classification.* 35K15, 35K55, 90A09, 91B28.

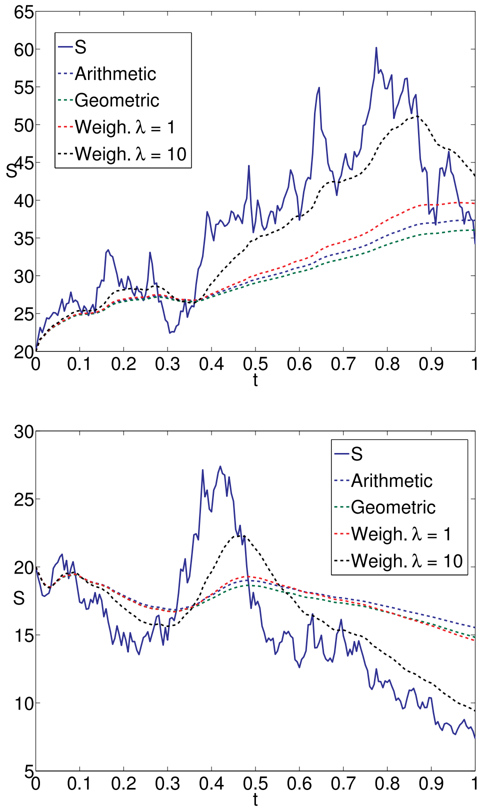

FIGURE 1. Examples of evolution of underlying asset prices (solid lines) and their path averages (dashed lines).

- arithmetic averaged options, where the average A_t^a is given by

$$A_t^a = \frac{1}{t}\int_0^t S_\xi \; d\xi, \tag{1}$$

- weighted arithmetic averaged options, with the average A_t^{wa} is given by

$$A_t^{wa} = \frac{1}{K(t)}\int_0^t a(t-\xi)S_\xi \; d\xi, \qquad \text{where} \;\; K(t) = \int_0^t a(\xi)d\xi, \tag{2}$$

  and a is an exponential weight function $a(\xi) = \exp(-\lambda\xi)$ with the averaging factor $\lambda > 0$,
- geometric averaged options, where the average A_t^g is given by

$$\ln A_t^g = \frac{1}{t}\int_0^t \ln S_\xi \; d\xi. \tag{3}$$

In Figure 1 we plot two different sample paths of the underlying asset price (solid lines) and their arithmetic, geometric and weighted arithmetic path averages. In the case of weighted arithmetic averaging with a weight factor $\lambda > 0$ we can observe that the average A_t^{wa} approaches the sample path S_t when $\lambda \to +\infty$. On the other hand, the weighted arithmetic average A_t^a approaches the arithmetic average when $\lambda \to 0^+$.