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SECOND-ORDER TOTAL VARIATION AND PRIMAL-DUAL ALGORITHM FOR CT IMAGE RECONSTRUCTION

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Abstract. In this paper, we proposed a regularization model based on second-order total variation for CT image reconstruction, which could eliminate the 'staircase' caused by total variation (TV) minimization. Moreover, some properties of second-order total variation were investigated, and a primal-dual algorithm for the proposed model was presented. Some numerical experiments for various projection data were conducted to demonstrate the efficiency of the proposed model and algorithm.

Key words. CT image reconstruction, regularization method, second-order total variation, primal-dual algorithm.

1. Introduction

Computed tomography (CT) is a noninvasive imaging technique, which plays an important role in modern medicine and industrial detections. CT image reconstruction methods have significant influence on the qualities of reconstructed images. These methods are mainly divided into two classes: analytical methods and algebraic methods [20, 27]. Generally speaking, the analytical methods, filtered back-projection for instance, are sensitive to noise, depend on the scan geometry and fail to deal with incomplete projection data. The algebraic methods, Kaczmarz method for example, are flexible for scan geometry and can deal with incomplete data partially, but suffer from high computation cost. However, algebraic methods attract increasingly attentions with the rapid development of computer technology. In this paper, we focus on the algebraic methods.

Approximating the unknown image by a 2D digital image $u \in \mathbb{R}^{m\times n}$ and denoting the intersection length of the i-th X-ray with the j-th pixel by a_{ij} (≥ 0), we can write the CT image reconstruction problem as to solve the following linear system [16, 20]

$$g = Au + \eta, \tag{1}$$

where $A = (a_{ij})_{M\times N}$ is called the imaging matrix, g is the projection data polluted by noise η, and u is the vector version of 2D image by lexicographic order. It is well known that recovering u from g by conventional direct methods is unfeasible due to the ill-posedness and large scale of A. Furthermore, neither analytical nor algebraic methods can handle incomplete (e.g. interior-CT) and heavily noised projection data.

Regularization techniques are important to reconstruct high quality image from incomplete and noised projection data. They are generally to minimize the following

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energy function [21, 30, 32, 33]

$$u^* = \arg\min_u \{F(u) + \lambda G(u)\} \ , \tag{2}$$

where F is called fidelity term measuring how fit Au is to the observation data g, and G is a convex function called regularization term representing prior knowledge. λ (> 0) is an user-defined parameter to balance the two terms.

We need to select proper regularization and fidelity terms for practical problems on hand. For CT image reconstruction problem (1), the fidelity term is usually chosen as l^2 distance since the noise η obeys Gaussian distribution [20]. For the regularization term, there are many selections, such as total variation (TV) method [31, 33, 34] and l^1 regularization based on wavelet or tight-frame technique [41, 42]. Although the dictionary learning based approaches were studied in the literature [24, 39] recently, they are not popular in the field of CT image reconstruction because of their time consuming training step and the requirement for high efficiency in the CT image reconstruction field, and TV methods are still the most widely used methods because of its edge-preserving properties and simplicity.

However, TV model often causes staircase effects (i.e. false edges) in smooth regions [3, 6]. The staircases are caused by the fact that the TV minimization forces the smooth regions (nonconstant) to be piecewise constant. Therefore, although the performances of TV model are amazing for numerical simulations on piecewise constant phantoms in the literature, it is not yet applied to clinical and related practice so far since few real CT images are piecewise constant [30].

In order to overcome the spurious staircases of TV, higher-order total variations (typically, second-order total variation) have been of particular interest and studied thoroughly over the past two decades [2, 9, 10, 25]. High-order TV (HOTV) was used to prove the uniqueness of interior-CT reconstruction if the region of interest (ROI) is piecewise polynomial [40]. However, the numerical computation of HOTV used in [40] is difficult. Therefore, we try applying second-order total variation (called SOTV for short) to CT image reconstruction.

The SOTV was first proposed in [25] to remove additive noise. It has been studied mainly to suppress the staircase effects of TV. The theoretical analyses in [17, 36] show that SOTV is superior to TV in some aspects. The SOTV evolves an observed image toward a 'smooth' one theoretically. The reconstructions of SOTV are believed to be better than those of TV in smooth region. This property had been verified by numerical experiments [10, 25] as well. In addition, SOTV can be numerical implementation more easily than HOTV [40]. As far as we know, although SOTV was studied thoroughly in the field of image restoration, it was rarely used in image reconstruction. In view of the discussions above, we propose to use the SOTV as the regularization term of (2), and our numerical experiments show that our model can suppress the 'staircase' effects in smooth regions effectively.

It is well known that fast and stable algorithm is crucial for the application of regularization model (2). Because of the non-smoothness of SOTV, a lot of algorithms (for example Newton method) are unfeasible. Recently, some algorithms have been proposed to tackle the non-smooth optimal problem, such as split Bregman iteration method [11, 15], primal-dual (PD) algorithm [5, 7, 43], alternative direction method of multiplier [29] and fixed-point algorithm [23, 26]. The PD algorithm, which was first proposed in [1], was investigated thoroughly [12, 18] and applied to solve the large scale problem in imaging science recently, such as image reconstruction [28, 38], restoration [37] and segmentation [8]. The PD algorithm is a general frame which can cover many models in image processing. In this paper,