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AN EFFICIENT APPROXIMATION FOR OPTIMAL DAMPING IN MECHANICAL SYSTEMS

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Abstract. This paper is concerned with an efficient algorithm for damping optimization in mechanical systems with a prescribed structure. Our approach is based on the minimization of the total energy of the system which is equivalent to the minimization of the trace of the corresponding Lyapunov equation. Thus, the prescribed structure in our case means that a mechanical system is close to a modally damped system. Although our approach is very efficient (as expected) for mechanical systems close to modally damped system, our experiments show that for some cases when systems are not modally damped, the proposed approach provides efficient approximation of optimal damping.

Key words. Linear vibration system, damped vibration, multi-variable optimization of dampers' viscosities, passive damping, Lyapunov equation, relative residual bound, linear residual bounds, quadratic residual bounds.

1. Nomenclature

We will use the following notation. Matrices written in the simple Roman fonts, M, D or K, for example, will have $\mathcal{O}(n^2)$ entries. Matrices written in mathematical bold fonts, $\mathbf{A}$, $\mathbf{B}$ will have $\mathcal{O}(m^2)$ entries, where $m = 2n$. The symbol $\|\cdot\|$ stands for the standard 2-norm.

2. Introduction

We consider a damped linear vibration system described by the differential equation

$$M\ddot{x} + D\dot{x} + Kx = 0,$$
$$x(0) = x_0, \quad \dot{x}(0) = \dot{x}_0,$$

where M, D, K (called mass, damping and stiffness matrix, respectively) are real, symmetric matrices of order n with M, K positive definite and $D = C_u + C$, where C_u is positive definite and presents internal damping, while C represents external damping and it is positive semidefinite. The matrix C_u is usually taken as a small multiple of critical damping or proportional damping. In this paper, we assume that internal damping is a small multiple of mass matrix, that is, $C_u = \alpha M$.

The problem of deriving *optimal damping* in some sense is an old and widely investigated problem which has been considered by many authors.

For example, in [1] the question of placement of damping elements was investigated, while in [2] the problem of periodic optimal control, which maximizes energy dissipation, was considered.

On the other hand, the optimization problem, which considers only viscosity optimization, was considered in the following papers [3], [4], [5], [6], [7] and [8].

In papers [9], [10] and [11], the authors have recemtly considered approximations based on modal eigenvectors which provide an efficient calculation of objective functions. The case of mechanical systems with a given force was considered in papers [12] and [13], where the authors derived explicit formulas for objective functions for particular types of mechanical systems, while in [14] it was shown how to compute eigenfrequencies of structures composed of a series of inclined cables.

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The purpose of this paper is to present new results of approximation algorithms for deriving optimal damping. As we will show, in some cases determination of optimal damping can be given by an explicit formula, while in some other cases we present a numerical approach to determination of optimal damping which can be efficiently implemented.

We are going to use optimization criterion considered in many papers, like [15],[3], [6], [5], [7]. This optimization criterion is given by the minimization requirement of the total energy of the system, that is,

$$\int_0^\infty E(t)\,dt \to \min\,. \tag{1}$$

Since criterion (1) depends on the initial condition, the simplest way to correct this is to take the average of (1) over all initial states of the unit total energy and a given frequency range. It can be shown that this average corresponds to the trace of the solution of the corresponding Lyapunov equation.

Since up to date an efficient general algorithm for the optimization of damping does not exist, that is, available algorithms optimize only viscosities of dampers, not their positions, we propose a simple and efficient approach to the overall damping optimization. With this new approach, one can find optimal positions and corresponding damper viscosities efficiently with satisfactory accuracy.

Our approach is based on the fact that for a modally damped mechanical system all three matrices M, D and K can be simultaneously diagonalized. Thus, the main assumption here will be that we have the case where M, D and K are simultaneously diagonalizable or that they are close to the case when all three matrices can be simultaneously diagonalized. Although this approach has been widely used by different scientific communities, especially in engineering, in this paper we propose a slightly different perspective, which will allow us to determine optimal damping very efficiently for a certain structure of mechanical systems, as will be demonstrated later.

Moreover, since only the damping matrix $D(v)$ depends on parameters, usual approaches to viscosity optimization (v) assume preprocessing based on diagonalization of the mass and stiffness matrices, M and K. On the other hand, in this paper we propose a new approach, which is based on diagonalization of the damping matrix $D(v)$, and then calculation of optimal viscosities. As we will show in this paper, this approach can be very efficient for structured systems which allow us to determine optimal viscosities, explicitly or numerically considerably faster.

For estimation of optimal viscosity for given damper positions we propose a new algorithm based on the simple "reduction" (truncation) of the corresponding Lyapunov equation, which usually speeds up the procedure by at least 40 times, while for the optimization of damper positions we propose a new heuristic. Both algorithms are based on a certain heuristic and unfortunately we do not have bounds for their accuracy, but as exemplified in the last section by the Lyapunov equation of modest dimensions ($n \leq 100$), they perform very well, thus we assume that the obtained results will be even better for bigger dimensions.

Currently, two types of algorithms are in use for the estimation of optimal viscosity (for given damper positions). The first type are the Newton-type algorithms for one-dimensional problems, which use some Lyapunov solvers, and the second type are the algorithms which explicitly calculate the trace of the solution of the corresponding Lyapunov equation.

Algorithms of the second type are presented in [15], [6] or [7], and they consider the case with one or more dampers with the same viscosity.

On the other hand, the Newton-type algorithm for the case with $r \geq 1$ different dampers was proposed in [16]. As shown in [7], the algorithm proposed in [16] can produce a poor result due to the problems with determination of the starting point.

The paper is organized as follows. In Section 2, we precisely define problem setting, while in Section 3, we present an approximation for our objective function. The problem of