Analysis in Theory and Applications
Anal. Theory Appl., Vol. **29**, No. 2 (2013), pp. 158-168
DOI: 10.4208/ata.2013.v29.n2.7

Fixed Point of Multivalued Operators on Partial Metric Spaces

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Received 2 May 2012

Abstract. In this paper, we prove some results on fixed point of multivalued operators on partial metric spaces.

Key Words: Fixed point, multivalued operator, partial metric space.
AMS Subject Classifications: 47H10, 54H25

1 Introduction

In 1992, Matthews introduced partial metric spaces as a generalization of the metric space [8]. In the partial metric space the distance of a point from itself is not necessarily zero [8]. Recently, these spaces have been considered by some authors [1, 2, 11]. There are a lot of fixed and common fixed point results in different types of spaces. After the remarkable contribution of Matthews, many authors have studied on partial metric spaces and its topological properties (see, e.g., [4–7, 10, 12, 13]. Then, Valero [14], Oltra and Valero [9] and Altun et al. [1] gave some generalizations of the result of Matthews. Also, Romaguera proved a Kirk type fixed point theorem on partial metric spaces [11].

A partial metric is a function $p: X\times X\to[0,\infty)$ satisfying the following conditions:

(a) $p(x,y)=p(y,x)$,

(b) If $p(x,x)=p(x,y)=p(y,y)$, then $x=y$,

(c) $p(x,x)\le p(x,y)$,

(d) $p(x,z)+p(y,y)\le p(x,y)+p(y,z)$,

for all $x,y,z\in X$. A space X with a partial metric p is called a partial metric space denoted by (X,p). If p is a partial metric p on X, then the function $d_p: X\times X\to[0,\infty)$ given by

$$d_p(x,y)=2p(x,y)-p(x,x)-p(y,y)$$

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is a metric on X. Also, each partial metric p on X generates a T_0 topology τ_p on X with a base of the family of open p-balls $\{B_p(x,\varepsilon): x\in X,\ \varepsilon>0\}$, where $B_p(x,\varepsilon)=\{y\in X: p(x,y)< p(x,x)+\varepsilon\}$ (see, e.g., [1,2,11]).

Definition 1.1 ([2,8])**.** Let (X,p) be a partial metric space.

(i) A sequence $\{x_n\}$ converges to $x\in X$ whenever $\lim_{n\to\infty} p(x_n,x)=p(x,x)$;

(ii) $\{x_n\}$ is called a Cauchy sequence if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists (and finite);

(iii) (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x\in X$, that is, $\lim_{n,m\to\infty} p(x_n,x_m)=p(x,x)$.

Definition 1.2. Let (X,p) be a partial metric space and $T:X\to X$ a selfmap. We say that T is orbitally continuous whenever $\lim_{n\to\infty} p(x_n,x)=p(x,x)$ implies that $\lim_{n\to\infty} p(Tx_n,Tx)= p(Tx,Tx)$.

We need the following results.

Lemma 1.1 ([2,8])**.** *Let (X,p) be a partial metric space. Then*

(a) *$\{x_n\}$ is a Cauchy sequence if and only if $\{x_n\}$ is a Cauchy sequence in the metric space (X,d_p);*

(b) *(X,p) is a complete partial metric space if and only if (X,d_p) is a complete metric space. Moreover,* $\lim_{n\to\infty} d_p(x,x_n)=0$ *if and only if*

$$\lim_{n\to\infty} p(x,x_n)=\lim_{n,m\to\infty} p(x_n,x_m)=p(x,x).$$

One can easily prove the next results.

Lemma 1.2. *Let (X,p) be a partial metric space. Then*

(a) *If $p(x,y)=0$, then $x=y$;*

(b) *If $x\neq y$, then $p(x,y)>0$.*

Lemma 1.3. *Let (X,p) be a partial metric space and $x_n\to z$ with $p(z,z)=0$. Then* $\lim_{n\to\infty} p(x_n,y)= p(z,y)$ *for all $y\in X$.*

For a partial metric space (X,p), let $CB(X)$ denote the collection of all nonempty, bounded and closed subsets of X. Let H_p be defined on $CB(X)$ by

$$H_p(A,B)=\max\Big\{\sup_{a\in A} p(a,B),\sup_{b\in B} p(b,A)\Big\},$$

where $p(x,A):=\inf\{p(x,y): y\in A\}$. Note that $p(x,Tx)\le p(x,y)+p(y,Tx)$ for all $x,y\in X$. In fact, for each $y\in X$ and $a\in Tx$ we have

$$p(x,a)\le p(x,y)+p(y,a)-p(y,y)\le p(x,y)+p(y,a)$$

and so $p(x,Tx)=\inf_{a\in Tx} p(x,a)\le p(x,y)+\inf_{a\in Tx} p(y,a)=p(x,y)+p(y,Tx)$. In this paper, we prove some results on fixed point of multivalued operators on partial metric spaces.