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CONVERGENCE OF A FINITE DIFFERENCE SCHEME FOR 3D FLOW OF A COMPRESSIBLE VISCOUS MICROPOLAR HEAT-CONDUCTING FLUID WITH SPHERICAL SYMMETRY

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Abstract. We consider the nonstationary 3D flow of a compressible viscous heat-conducting micropolar fluid in the domain to be a subset of $\mathbf{R^3}$, bounded with two concentric spheres. In the thermodynamical sense the fluid is perfect and polytropic. The homogeneous boundary conditions for velocity, microrotation, heat flux and spherical symmetry of the initial data are proposed. Due to the assumption of spherical symmetry, the problem can be considered as one–dimensional problem in Lagrangian description on the domain that is a segment. We define the approximate equations system by using the finite difference method and construct the sequence of approximate solutions for our problem. By analyzing the properties of these approximate solutions we prove their convergence to the generalized solution of our problem globally in time and establish the convergence of the defined numerical scheme, which is the main result of the paper. The practical application of the proposed numerical scheme is performed on the chosen test example.

Key words. micropolar fluid flow; spherical symmetry; finite difference approximations; strong and weak convergence.

1. Introduction

The theory of micropolar fluids, established by Eringen [13], provides a mathematical foundation for studying the model of a fluid, which takes into account the interactions between the micromotion effects of fluid particles and the macromotion. Eringen's theory has provided a good model to study a fluid flow in which the influence of the microstructure on the flow itself is not negligible. The examples of such fluids are polymeric suspensions, biological fluids, liquid cristals, biofluidics, muddy fluids, etc. The application of these fluids are in blood flow, lubrication theory, flow in capillares and microchannels, etc. Due to a number of practical applications (see [14, 19]), in recent years, the model has become an important area of interest for theoretical and applied mathematicians [6, 16, 26, 29, 5], as well as, for the engineers [17, 18, 27]. The majority of work considering the micropolar fluids refers to the incompressible flow. The theory and known results from mathematical point of view are very well systematized in the book of Lukaszewicz [19], but there are still many open problems. The compressible flow of the micropolar fluid has begun to be intensively studied in the last few years [6, 16, 29, 5].

In this paper we focus on the compressible flow of the isotropic, viscous and heat conducting micropolar fluid, which is in the thermodynamical sense perfect and polytropic. We consider the fluid in the domain $\Omega = \{\mathbf{x} \in \mathrm{R}^3 |\ a < |\mathbf{x}| < b\}$, where $|\mathbf{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2}$, $b > a > 0$ and assume that the initial data are spherically symmetric. The homogeneous boundary conditions for velocity, microrotation and heat flux are proposed. Taking into account the spherical symmetry of our problem, we get one-dimensional problem, which is analysed here in the Lagrangian description.

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The model for this type of flow was first considered by Mujaković in [20] where she developed a one-dimensional case. In the same work, the local existence and uniqueness of the solution, which is called generalized, for the model with the homogeneous boundary conditions are proved. Mujaković in [21] and in the references cited therein proved the local and global existence for the same one-dimensional model with the nonhomogeneous boundary conditions for velocity and microrotation, as well as the stabilization and the regularity of the solution. In [22] the Cauchy problem for this one-dimensional problem was also considered.

The described model of compressible micropolar fluid in the three-dimensional case was first considered in [9]. Assuming the homogeneous boundary conditions and spherical symmetry, the existence and uniqueness of the generalized solution locally in time was proved by using the Faedo-Galerkin method. Applying this result, with the help of the extension principle, the existence of the generalized solution globally in time was proved in [11].

The main goal of this paper is to apply the finite difference method to the described micropolar fluid flow problem and to prove its convergence. The method is based on the defined finite difference approximate equations system and its convergence is established by analysing the properties of the sequence of approximate solutions. We prove that the limit of this sequence is the solution to our problem with the same properties as the solutions obtained in [9] and [11]. In this way, the global existence of generalized solution is established again, but now by using the finite difference method. Let us mention that this proof is technically more demanding that the already obtained proof in which the Faedo-Galerkin method, together with the extension principle, was used [9, 11], but it has the methodological advantages as follows. First, we do not need the local existence theorem for the proof of the global existence. Second, the Faedo-Galerkin method is limited to the problems with the smooth enough initial functions and to the problems with homogeneous boundary conditions, while the method from this work could be extended to other classes of initial functions, for example, to the initial functions with discontinuities (see [3]) and to the problems with non-homogeneous boundary conditions. Thus, on the one hand, the paper could be of interest to theoretical mathematicians working in the area of compressible flows, since it offers a possible approach to the proof of the existence of global solutions to similar types of problems. On the other hand, the paper provides the convergent numerical scheme, which could be of interest to the applied mathematicians and engineers when performing numerical simulations for the considered type of problems.

The procedure used in this work is similar to the procedure applied in [25], where we proved the existence of the solution globally in time for the model that governs one-dimensional flow. However, in comparison with [25], the mathematical model used in this work is more complex because it contains function $r(x,t)$ that represents the Eulerian coordinate, which refers to the Lagrangian coordinate x in time t. As a consequence, we need to derive more a priori estimates for approximate solution, including the estimates for approximations of the function r, for which we have to show the strong convergence.

In our research, there is an important influence of the results from [4], where the existence of the solution globally in time for the three-dimensional spherically symmetric model of the classical fluid flow (with microrotation equals zero) is proved by using the finite difference method. We follow some ideas from [3] also.