INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 12, Number 1, Pages 54–80

STABILITY OF THE KINEMATICALLY COUPLED β-SCHEME FOR FLUID-STRUCTURE INTERACTION PROBLEMS IN HEMODYNAMICS

SUNČICA ČANIĆ, BORIS MUHA, AND MARTINA BUKAČ

(Communicated by Max Gunzburger)

Abstract. It is well-known that classical Dirichlet-Neumann loosely coupled partitioned schemes for fluid-structure interaction (FSI) problems are unconditionally unstable for certain combinations of physical and geometric parameters that are relevant in hemodynamics. It was shown in [18] on a simple test problem, that these instabilities are associated with the so called "added-mass effect". By considering the same test problem as in [18], the present work shows that a novel, partitioned, loosely coupled scheme, recently introduced in [11], called the kinematically coupled β-scheme, does not suffer from the added mass effect for any $\beta \in [0, 1]$, and is unconditionally stable for all the parameters in the problem. Numerical results showing unconditional stability are presented for a full, nonlinearly coupled benchmark FSI problem, first considered in [31].

Key words. Fluid-structure interaction, Partitioned schemes, Stability analysis, Added-mass effect

1. Introduction

Fluid-structure interaction (FSI) problems have important applications in various areas including bio-fluids and aero-elasticity. They have been extensively studied from the numerical, as well as analytical point of view [5, 7, 8, 9, 10, 19, 20, 21, 23, 24, 28, 30, 32, 36, 38, 39, 41, 42, 48, 49, 50, 55, 56]. A set of popular numerical schemes for FSI in blood flow includes partitioned schemes (loosely or strongly coupled). Partitioned schemes typically solve an underlying multi-physics problem by splitting the problem into sub-problems determined by the different physics in the coupled problem. In particular, in fluid-structure interaction problems the fluid dynamics and structure elastodynamics are often solved using separate solvers. In loosely coupled schemes only one iteration between the fluid and structure sub-problem is performed at each time step, while in strongly coupled schemes several sub-iterations between the fluid and structure sub-problems need to be performed at each time step to achieve stability.

The main advantages of *loosely coupled partitioned schemes* are modularity, simple implementation, and low computational costs. However, in [18] it was proved that for certain combinations of physical and geometric parameters (which are realistic in blood flow) "classical" Dirichlet-Neumann loosely coupled schemes are unconditionally unstable. In the same paper, the authors showed that this instability is due to the "added-mass effect". Namely, it was shown that a portion of the fluid load to the structure in the coupled FSI problems can be written as an additional inertia term in the structure equation coming from the fluid mass (added mass). In numerical schemes in which this term appears *explicitly*, which is the case in the classical Dirichlet-Neumann loosely coupled partitioned schemes, the

Received by the editors May 10, 2013 and, in revised form, April 23, 2014.
2000 *Mathematics Subject Classification.* 35R35, 49J40, 60G40.
Corresponding author: Sunčica Čanić.

added mass term acts as a source of instabilities when the structure is too "light" to counter-balance the kinetic energy of the "heavy" fluid load.

To get around these difficulties, several different loosely coupled algorithms have been proposed. The method proposed in [5] uses a simple membrane model for the structure which can be easily embedded into the fluid problem where it appears as a generalized Robin boundary condition. In this way the original problem reduces to a sequence of fluid problems with a generalized Robin boundary condition which can be solved using only the fluid solver. A similar approach was proposed in [48] where the fluid and structure were split in the classical way, but the fluid and structure sub-problems were linked via novel transmission (coupling) conditions that improve the convergence rate. A different approach to stabilization of loosely coupled (explicit) schemes was proposed in [16] based on the Nitsche's method [35]. We further mention the scheme proposed in [6] where a Robin-Robin type preconditioner was combined with Krylov iterations for a solution of an interface system.

For completeness, we also mention several semi-implicit FSI schemes. The schemes proposed in [27, 1, 2] separate the computation of fluid velocity from the coupled pressure-structure velocity system, thereby reducing the computational costs. Similar schemes, derived from algebraic splitting, were proposed in [3, 52]. We also mention [46] where an optimization problem is solved at each time-step to achieve continuity of stresses and continuity of velocity at the interface.

Recently, a novel loosely coupled partitioned scheme, called the "kinematically coupled β-scheme", was introduced in [11]. Because of its simple implementation, modularity, and good performance, the kinematically-coupled scheme and its modifications provide an appealing way to study multi-physics problems involving FSI. Indeed, this scheme has been used by several groups to study FSI problems in hemodynamics including poroelastic arterial walls [15], non-Newtonian fluids [40], cardiovascular stents [44], thin structures with longitudinal displacement [12, 45], FSI with thick structures [13], and FSI with multi-layered structure of arterial walls [43, 14]. See also [25, 29, 26] for a generalization of this scheme called "the incremental displacement-correction scheme." The kinematically-coupled β-scheme successfully deals with problems associated with the added mass effect in a way different from those reported above. It is a modification of the kinematically coupled scheme first introduced in [34]. The parameter β was introduced in [11] to increase the accuracy. This parameter distributes the fluid pressure between the fluid and structure sub-problems.

In [42] the authors used the kinematically-coupled scheme to prove the existence of a weak solution to a fully nonlinear FSI problem between an incompressible, viscous fluid and a thin structure modeled by either the elastic or viscoelastic shell equations. The existence proof is based on constructing approximate solutions using the kinematically-coupled scheme, and showing that the approximate solutions converge to a weak solution as the time-discretization tends to zero. This existence result relies on energy estimates, which show that in the kinematically-coupled scheme the energy of the coupled FSI problem is well-approximated by the discretized problem. A consequence of this result is that the kinematically-coupled scheme is stable.

In contrast with the energy estimates approach showing unconditional stability of the scheme, the present manuscript uses a different approach, similar to that of [18] (which was based on the Dirichlet-to-Neumann mapping applied to a simplified FSI problem), which reveals, explicitly, how and why this scheme is unconditionally stable for any $0 \leq \beta \leq 1$.