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Review and Study of Some Doubly-Weighted Pseudo-almost Periodic Solutions of Some Partial Evolution Equations

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Abstract. In this work, a characterization of the doubly-weighted Bohr spectrum of an almost periodic function is given. We showed precisely that such a spectrum is either empty or coincides with the Bohr spectrum of that function. Next, we investigate the existence of doubly-weighted pseudo-almost periodic solutions to some classes of non-autonomous partial evolution equations.

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1 Introduction

In [1], results on the existence and uniqueness are provided for doubly-weighted pseudo-almost periodic solution for equations

$$u'(t)=A(t)u(t)+g(t,u(t)),\quad t\in\mathbb{R}, \tag{1.1}$$

where $A(t)$ for $t\in\mathbb{R}$ is a family of closed linear operators on $D(A(t))$ satisfying the well-known Acquistapace-Terreni conditions, and $g:\mathbb{R}\times\mathbb{X}\to\mathbb{X}$ is doubly-weighted pseudo-almost periodic in $t\in\mathbb{R}$ uniformly in the second variable.

The existence of weighted pseudo-almost periodic, weighted pseudo-almost automorphic, and pseudo-almost periodic solutions to differential equations constitutes one

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of the most attractive topics in qualitative theory of differential equations due to possible applications. Some contributions on weighted pseudo-almost periodic functions, their extensions, and their applications to differential equations have recently been made, among them are for instance [2–17] and the references therein. However, the problem which consists of the existence of doubly-weighted pseudo-almost periodic(mild) solutions to evolution equations in the form (1.1) is quite new and untreated and thus constitutes one of the main motivations of the theme. In the present work, we consider a more general setting and use slightly different techniques to study the existence of doubly-weighted pseudo-almost periodic solutions to the following classes of partial evolution equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big[u(t)+f(t,B(t)u(t))\Big]=A(t)u(t)+g(t,C(t)u(t)), \qquad t\in\mathbb{R}, \tag{1.2}$$

where $A(t)$ for $t\in\mathbb{R}$ is a family of closed linear operators on $D(A(t))$ satisfying the well-known Acquistapace-Terreni conditions, $B(t),C(t)$ $(t\in\mathbb{R})$ are families of (possibly unbounded) linear operators, and $f:\mathbb{R}\times\mathbb{X}\mapsto\mathbb{X}^t_\beta$, $g:\mathbb{R}\times\mathbb{X}\mapsto\mathbb{X}$ are doubly-weighted (μ,ν)-pseudo almost periodic in $t\in\mathbb{R}$ uniformly in the second variable.

The paper is organized as follows: Section 2 is devoted to preliminary results related to the existence of an evolution family, intermediate spaces, properties of weights, and basic definitions and results on the concept of doubly-weighted pseudo-almost periodic functions. In Section 3 we study the existence of a doubly-weighted mean for almost periodic functions. In Section 4 we prove the existence of doubly-weighted pseudo-almost periodic solutions to (1.2). In Section 5, we give an application to illustrate our results.

2 Preliminaries

2.1 Evolution family and exponential dichotomy

In this section, we introduce the inter and extrapolation spaces for $A(t)$. Consider a family of closed linear operators $A(t)$ for $t\in\mathbb{R}$ on $\mathbb{X}$ with domain $D(A(t))$ satisfying the so-called Acquistapace-Terreni conditions:

H_0: $\exists\,\omega\in\mathbb{R}$, $\theta\in(\frac{\pi}{2},\pi)$, $K,L\geq 0$ and $\mu,\nu\in(0,1]$ with $\mu+\nu>1$ such that

$$\Sigma_\theta\cup\{0\}\subset\rho\Big(A(t)-\omega\Big)\ni\lambda, \Big\|R(\lambda,A(t)-\omega)\Big\|\leq\frac{K}{1+|\lambda|}, \tag{2.1}$$

$$\Big\|\Big(A(t)-\omega\Big)R\Big(\lambda,A(t)-\omega\Big)\Big[R\Big(\omega,A(t)\Big)-R\Big(\omega,A(s)\Big)\Big]\Big\|\leq L\frac{|t-s|^\mu}{|\lambda|^\nu}, \tag{2.2}$$

for $t,s\in\mathbb{R}$, $\lambda\in\Sigma_\theta:=\{\lambda\in\mathbb{C}\setminus\{0\}:|\arg\lambda|\leq\theta\}$.

For a given family of linear operators $A(t)$, the existence of an evolution family associated with it is not always guaranteed. However, if $A(t)$ satisfies Acquistapace-Terreni,