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Infinite Sequence Solutions for Space-Time Fractional Symmetric Regularized Long Wave Equation

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Abstract. In this paper, we investigate the space-time fractional symmetric regularized long wave equation. By using the Bäcklund transformations and nonlinear superposition formulas of solutions to Riccati equation, we present infinite sequence solutions for space-time fractional symmetric regularized long wave equation. This method can be extended to solve other nonlinear fractional partial differential equations.

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Key Words: Space-time fractional symmetric regularized long wave equation; Bäcklund transformations; nonlinear superposition formulas; exact solutions.

1 Introduction

In the past decades, much effort has been devoted to study nonlinear partial differential equations [1-25]. Nonlinear fractional partial differential equations (NFPDEs) regarded as the generalization form of nonlinear partial differential equations of integer order have attracted considerable attention in recent years. Moreover, the investigation of exact and approximate solutions for NFPDEs arising in mathematical physics, chemistry, biology, engineering, control theory, signal processing and so forth has become one of the most active and important research areas. A variety of analytical and numerical techniques have been well established and applied to solve NFPDEs, including the homogeneous balance method [6], the fractional sub-equation method [7-11], the exp-function method [12], the (G'/G)-expansion method [13, 14], the first integral method [15], the modified trial equation method [16], the Jacobi elliptic equation method [17], the modified

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Kudryashov method [18], the homotopy analysis transform method [19], the fractional variational iteration method [20], the Adomian decomposition method [21], and so on. In many analytical methods, the fractional complex transformation proposed by Li and He [22] plays a key role in converting NFPDEs into NODEs. The purpose of present article is to examine the space-time fractional symmetric regularized long wave (FSRLW) equation by means of Riccati equation method and symbolic computation. As a result, based on the Bäcklund transformations and nonlinear superposition formulas of solutions to Riccati equation, infinite sequence solutions in terms of trigonometric and hyperbolic functions are established.

For the convenience of a reader, we recall the Jumarie's modified Riemann-Liouville derivative [23] of order α, that is

$$D_t^\alpha f(t)=\begin{cases}\dfrac{1}{\Gamma(-\alpha)}\displaystyle\int_0^t (t-\xi)^{-\alpha-1}(f(\xi)-f(0))\mathrm{d}\xi, & \alpha<0,\\ \dfrac{1}{\Gamma(1-\alpha)}\dfrac{\mathrm{d}}{\mathrm{d}t}\displaystyle\int_0^t (t-\xi)^{-\alpha}(f(\xi)-f(0))\mathrm{d}\xi, & 0<\alpha<1,\\ \left(f^{(n)}(t)\right)^{(\alpha-n)}, \qquad n\le\alpha<n+1,\ n\ge 1. \end{cases}$$

Some significant properties of fractional modified Riemann-Liouville derivative are

$$\begin{aligned}&D_t^\alpha t^\delta=\frac{\Gamma(1+\delta)}{\Gamma(1+\delta-\alpha)}t^{\delta-\alpha}, \qquad \delta>0,\\ &D_t^\alpha(f(t)g(t))=g(t)D_t^\alpha f(t)+f(t)D_t^\alpha g(t),\\ &D_t^\alpha f[g(t)]=f_g'[g(t)]D_t^\alpha g(t)=D_g^\alpha[g(t)](g'(t))^\alpha.\end{aligned}$$

The layout of this paper is as follows. In Section 2 and Section 3, we present the main steps of Riccati equation method, and list the Bäcklund transformations and nonlinear superposition formulas [24, 25] of solutions to Riccati equation. In Section 4, we apply this method to establish infinite sequence solutions for space-time FSRLW equation. The last section is the conclusion.

2 Method

Consider a NFPDE in three independent variables as

$$P\big(u,\, D_t^\alpha u,\, D_x^\beta u,\, D_y^\gamma u,\, D_t^\alpha D_t^\alpha u,\, D_t^\alpha D_x^\beta u,\, \cdots\big)=0, \qquad 0<\alpha,\, \beta,\, \gamma\le 1, \tag{2.1}$$

where $D_t^\alpha u$, $D_x^\beta u$, $D_y^\gamma u$, $\cdots$ are the modified Riemann-Liouville derivatives, and P is a polynomial in u and its fractional derivatives. We find solutions to Eq. (2.1) in the form

$$u(t,\, x,\, y)=U(\xi),\ \xi=\frac{ct^\alpha}{\Gamma(1+\alpha)}+\frac{kx^\beta}{\Gamma(1+\beta)}+\frac{ly^\gamma}{\Gamma(1+\gamma)}.$$