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Random Attractor for the Nonclassical Diffusion Equation with Fading Memory

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Abstract. In this paper, we consider the stochastic nonclassical diffusion equation with fading memory on a bounded domain. By decomposition of the solution operator, we give the necessary condition of asymptotic smoothness of the solution to the initial boundary value problem, and then we prove the existence of a random attractor in the space $\mathcal{M}_1=\mathcal{D}(A^{\frac{1}{2}})\times L^2_\mu(R^+,\mathcal{D}(A^{\frac{1}{2}}))$, where $A=-\Delta$ with Dirichlet boundary condition.

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1 Introduction

In this paper, we use the method described in [1–3] to consider a stochastic nonclassical diffusion equation with fading memory on a bounded domain:

$$\begin{cases} u_t-\Delta u_t-\Delta u-\displaystyle\int_0^\infty \kappa(s)\Delta u(t-s)\mathrm{d}s=f(u)+g(x)+h(x)\frac{\mathrm{d}W}{\mathrm{d}t}, & x\in D,t>0,\\ u(x,t)=0, \quad x\in\partial D, & \\ u(x,t)=u_0(x,\tau), \quad x\in D,\tau\le 0, & \end{cases} \tag{1.1}$$

where D is a bounded domain in $R^n(n\ge 3)$, $h(x)\in H^1_0(D)\bigcap H^2(D)$, $W(t)$ is a two-sided real valued Wiener process on the probability space $(\Omega,\mathcal{F},P)$. About the forcing term g and the nonlinearity f, we assume that $g(x)\in H^{-1}(D)$, f is a Lipschitz continuous function and satisfies that

$$|f(s)|\le\alpha_2|s|^{p-1},\quad f(s)s\le-\alpha_1|s|^p,\quad |f'(s)|\le C(1+|s|^{\frac{4}{n-2}}),\quad f(0)=0, \tag{1.2}$$

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for $s\in R$ and $2\leq p\leq 2n/(n-2)$.

This problem has its origin in the mathematical description of viscoelastic materials. It is well known that the viscoelastic materials exhibit natural damping, which is due to the special property of these materials to retain a memory of their past history. And from the materials point of view, the property of memory comes from the memory kernel $\kappa(s)$, which decays to zero with exponential rate, so it is the fading memory of the far history in the model.

Many authors have studied the classical reaction diffusion equations [3–8], but it doesn't contain all the aspects of the reaction diffusion problem. K. Kuttler and E.C. Aifantis [9] proved the existence and uniqueness of the solution for the nonclassical diffusion equations $u_t-\Delta u_t-\Delta u=f(u)+g(x)$, and Q. Z. Ma [10] study the existence of global attractors for nonclassical diffusion equations in $H^1(R^N)$ with the nonlinearity satisfies the arbitrary order polynomial growth conditions. In [3,11–14], attractors for the wave equations or nonclassical diffusion equations with the memory kernel have been proved. N. Tatar [1] and Wu, Liu et al [15,16]considered the exponential stability for the wave equation with a temporal non-local term and asymptotic behavior of solutions for the wave equations with memory.

In recent years, the existence and uniqueness of the solutions and the long time dynamics for the stochastic reaction diffusion equation have been proved, which contain bounded and unbounded domains [17–20].

In this paper, suppose that the memory kernel

$$\kappa(s)\in C^2(R^+),\quad \kappa(s)\geq 0,\quad \kappa'(s)\leq 0,\qquad \forall s\in R^+,$$

and assume that there exists a positive constant $\delta>0$ such that the function $\mu(s)=-\kappa'(s)$ satisfies

$$\mu\in C^1(R^+)\cap L^1(R^+),\quad \mu'(s)\leq 0,\quad \mu'(s)+\delta\mu(s)\leq 0,\qquad \forall s\geq 0. \tag{1.3}$$

To our knowledge, Eq. (1.1) has not been considered by predecessors and is studied firstly as a new model in this paper. Since Eq. (1.1) contains memory term, we first construct relatively complicate solution space and make a priori estimate in the space. Simultaneously, it is also difficult to test and verify the property of the solution semigroup such as continuity, compactness, or asymptotic compactness. These obstacles possibly lead to various additional conditions which seem unnecessary on terms of (1.1)(cf. [21]).

This paper is organized as follows.In the next section, we recall some preliminaries, including the notation that we will use, some preliminary results related to the solution of the stochastic equation and the random attractor for random dynamical systems. In Section 3, firstly, we define a continuous random dynamical system for prove the existence and uniqueness of the solution for the stochastic nthe nonclassical diffusion equation with fading memory, then prove exist a closed random absorbing and establish the asymptotic compactness of the random dynamical system and prove the existence of a random attractor.

Throughout this paper, we denote C is a constant from line to line.