JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **27**, No. 4, pp. 283-292

doi: 10.4208/jpde.v27.n4.1
December 2014

Non-local Boundary Value Problem for the Third Order Mixed Type Equation in Double-connected Domain

ABDULLAYEV O. Kh*

National University of Uzbekistan, Tahskent 700-174, Uzbekistan.

Received 7 March 2014; Accepted 31 August 2014

Abstract. In the present paper an existence and uniqueness of solution of the nonlocal boundary value problem for the third order loaded elliptic-hyperbolic type equation in double-connected domain have been investigated. At the proof of unequivocal solvability of the investigated problem, the extremum principle for the mixed type equations and method of integral equations have been used.

AMS Subject Classifications: 35M10

Chinese Library Classifications: O175.2

Key Words: Loaded equation; elliptic-hyperbolic type; double-connected domain; an extremum principle; existence of solution; uniqueness of solution; method of integral equations.

1 Introduction

Boundary value problems (BVP) for mixed type equations have numerous applications in physics, biology and in other material sciences. F. I. Frankl [1] found critical applications of the problem Tricomi and others related problems in transonic gas dynamics. In the works by A. V. Bitsadze [2], the analogue of problem Tricomi for the first time is formulated and investigated in double-connected domain for the equations of the mixed type, which was new direction of the theory partial differential equations. After this work various problems for the equation of the mixed elliptic-hyperbolic type of the second order in double connected domain were investigated. For example, we note works by M. S. Salahitdinov and A. K. Urinov [3], B. Islomov and O. Kh. Abdullaev [4] and etc.

We as well note that with intensive research on problem of optimal control of the agroeconomical system, regulating the label of ground waters and soil moisture, it has become necessary to investigate a new class of equations called "LOADED EQUATIONS".

*Corresponding author. *Email addresses:* obidjon.mth@gmail.com (O. Kh. Abdullayev)

Such equations were investigated in first in the works of N. N. Nazarov and N. Kochin, but they didn't use the term "LOADED EQUATIONS". For the first time was given the most general definition of a LOADED EQUATIONS and various loaded equations are classified in detail by A. M. Nakhushev [5]. After this work has appeared very interesting results of the theory of boundary value problems for the loaded equations parabolic, parabolic-hyperbolic and elliptic-hyperbolic types. We note works of: A. M. Nakhushev [6], K. B. Sabitov and E. R. Melisheva [7], E. R. Melisheva [8] and O. Kh. Abdullaev [9], [10].

Local and non-local problems for the third order loaded equations elliptic-hyperbolic type in double-connected domains as well were not investigated.

In the present paper, the uniqueness and the existence of the solution of non-local boundary value problem for the third order loaded equation of elliptic-hyperbolic type in double-connected domain was proved.

2 The statement of the problem

Consider the equation

$$\frac{\partial}{\partial y}\left[u_{xx}+sgn(xy)u_{yy}+\lambda\Re(x,y)u(0,y)\right]=0, \tag{2.1}$$

in double-connected domain Ω, bounded with two lines:

$$\sigma_1: x^2+y^2=1, \qquad \sigma_2: x^2+y^2=q^2, \qquad \text{at } x>0, y>0;$$
$$\sigma_1^*: x^2+y^2=1, \qquad \sigma_2^*: x^2+y^2=q^2, \qquad \text{at } x<0, y<0,$$

and characteristics: $A_jA_j^*: x-y=(-1)^{j+1}$, $B_jB_j^*: x-y=(-1)^{j-1}q$, $(j=1,2)$ at $xy<0$ of the Eq. (2.1), where

$$\Re(x,y)=\frac{1-\text{sgn}(xy)}{2}\cdot\frac{1+\text{sgn}(y^2+xy)}{2}, \quad \lambda=\text{const}>0,\ A_1(1;0),\ A_2(0;1),\ A_1^*(0;-1),$$
$$A_2^*(-1;0),\ B_1(q;0),\ B_2(0;q),\ B_1^*(0;-q),\ B_2^*(-q;0),\ (0<q<1).$$

We will introduce designations:

$$E_j\left(\frac{q-(-1)^j}{2};\frac{q+(-1)^j}{2}\right), \qquad E_j^*\left(\frac{(-1)^{j-1}-q}{2};\frac{(-1)^j-q}{2}\right),$$
$$\Omega_0=\Omega\cap(x>0)\cap(y>0),\ \Omega_0^*=\Omega\cap(x<0)\cap(y<0),\ \Delta_1=\Omega\cap(x+y>q)\cap(y<0);$$
$$\Delta_1^*=\Omega\cap(x+y<-q)\cap(y<0),\ \Delta_2=\Omega\cap(x+y>q)\cap(x<0),$$
$$\Delta_2^*=\Omega\cap(x+y<-q)\cap(y>0),\ D_1=\Omega\cap(-q<x+y<q)\cap(y<0),$$
$$D_2=\Omega\cap(-q<x+y<q)\cap(y>0),\ D_0=\Omega_0\cup\Delta_1\cup\Delta_2,\ D_0^*=\Omega_0^*\cup\Delta_1^*\cup\Delta_2^*,$$
$$I_j=\{t: 0<t<q\},\ I_{2+j}=\left\{t: 0<(-1)^{j-1}t<1\right\},\ (j=1,2),\ \text{where } t=\begin{cases} x;\ j=1,\\ y;\ j=2,\end{cases}$$