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Blow Up of Solutions to One Dimensional Initial-Boundary Value Problems for Semilinear Wave Equations with Variable Coefficients

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Abstract. This paper is devoted to studying the following initial-boundary value problem for one-dimensional semilinear wave equations with variable coefficients and with subcritical exponent:

$$u_{tt}-\partial_x\big(a(x)\partial_x u\big)=|u|^p,\quad x>0,\quad t>0,\quad n=1, \tag{0.1}$$

where $u=u(x,t)$ is a real-valued scalar unknown function in $[0,+\infty)\times[0,+\infty)$, here $a(x)$ is a smooth real-valued function of the variable $x\in(0,+\infty)$. The exponents p satisfies $1<p<+\infty$ in (0.1). It is well-known that the number $p_c(1)=+\infty$ is the critical exponent of the semilinear wave equation (0.1) in one space dimension (see for e.g., [1]).

We will establish a blowup result for the above initial-boundary value problem, it is proved that there can be no global solutions no matter how small the initial data are, and also we give the lifespan estimate of solutions for above problem.

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1 Introduction and main results

In one dimensional case, initial-boundary value problems for one dimensional semilinear wave equations on exterior domain is reduced to the initial-boundary value problem

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on the semi-infinite interval $[0,+\infty)$. In this paper, we will consider the blow up of solutions of the following initial-boundary value problem with small initial data and zero boundary data for the following one dimensional semilinear wave equations:

$$\begin{cases} u_{tt}-\partial_x\big(a(x)\partial_x u\big)=|u|^p, & x>0, \quad t>0, \quad n=1, \\ u(0,x)=\varepsilon f(x), \quad u_t(0,x)=\varepsilon g(x), & x>0, \\ u|_{x=0}=0, & t\geq 0, \end{cases} \tag{1.1}$$

where $a(x)$ is a smooth function of the variable $x\in(0,+\infty)$, which takes values in the real, such that for some $C\geq 1$,

$$C^{-1}\leq a(x)\leq C, \quad \forall x>0,$$

and

$$a(x)=1, \quad \text{when } \ x\geq R.$$

Without loss of generality, we assume that $\text{supp}\{f,g\}\subset\{x|0<x\leq R\}$. We consider dimension $n=1$ and exponents $p\in(1,+\infty)$ for problem (1.1), where $p_c(1)=+\infty$. The number $p_c(1)=+\infty$ is known as the critical exponent of the one-dimensional semilinear wave equation (1.1) (see, e.g., [1]). And we consider compactly supported nonnegative data $(f,g)\in H^1((0,+\infty))\times L^2((0,+\infty))$ for problem (1.1).

Generally, when $n=1$, let $p_c(1)=+\infty$; and when $n\geq 2$, let $p_c(n)$ is the positive root of the quadratic equation

$$(n-1)p^2-(n+1)p-2=0,$$

the number $p_c(n)$ is known as the critical exponent of semilinear wave equations, since it divides $(1,+\infty)$ into two subintervals so that the following take place: If $p\in(1,p_c(n))$, then solutions with nonnegative initial values blow up in finite time; if $p\in(p_c(n),+\infty)$, then solutions with small (and sufficiently regular) initial values exist for all time (see for e.g., [1]). The proof has an interesting and exciting history that spans three decades. We only give a brief summary here and refer the reader to [1, 2] and the references therein for details. The problem about existence or nonexistence of global solutions is sometimes referred to as the Conjecture of Strauss [3]. The question was also asked by Glassey [4].

If $a(x)=1$, we say problems (1.1) are of constant coefficients. In the case of cauchy problems of semilinear wave equation with constant coefficients in all space dimensions, there is an extensive literature which we shall review briefly, for details, see [5–20].

Recently, Y. Zhou and W. Han [21] has established the blow-up results and also obtained the upper estimate of the lifespan to solutions for the initial-boundary value problem (1.1) with $1<p<p_c(n)$ and the higher-dimensional case $n\geq 3$. And the author in [22] has established the blow-up results and the upper bound estimate of the lifespan to solutions in two dimensional case. However, up to now, to the best of our knowledge, there is no blowup result concerning initial-boundary value problems for one dimensional semilinear wave equations with variable coefficients on exterior domain. This paper attempts to fill this gap in the literature. In this paper, we shall establish blowup results for the one-dimensional initial-boundary value problem for subcritical values of p. We shall