JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **24**, No. 1, pp. 15-36

doi: 10.4208/jpde.v24.n1.2
February 2011

Synchronization of Stochastic Two-Layer Geophysical Flows

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Received 4 November 2009; Accepted 23 November 2010

Abstract. In this paper, the two-layer quasigeostrophic flow model under stochastic wind forcing is considered. It is shown that when the layer depth or density difference across the layers tends to zero, the dynamics on both layers synchronizes to an averaged geophysical flow model.

AMS Subject Classifications: 60H15, 37H10, 86A05, 76U05, 34D35

Chinese Library Classifications: O175.29

Key Words: Stochastic flow models; random dynamical systems; synchronization; stochastic PDEs; geophysical and climate dynamics.

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1 Introduction

We consider the two-layer quasigeostrophic flow model ([1], p. 423; [2], p. 87):

$$\frac{\partial q_1}{\partial t}+J(\psi_1,q_1+\beta y)=\nu\Delta^2\psi_1+f(x,y)+\dot{W}_1(t,x,y), \tag{1.1a}$$

$$\frac{\partial q_2}{\partial t}+J(\psi_2,q_2+\beta y)=\nu\Delta^2\psi_2-r\Delta\psi_2+\dot{W}_2(t,x,y), \tag{1.1b}$$

where potential vorticities $q_1(x,y,t)$, $q_2(x,y,t)$ for the top layer and the bottom layer are defined via stream functions $\psi_1(x,y,t)$, $\psi_2(x,y,t)$, respectively,

$$\begin{aligned} q_1&=\Delta\psi_1-F\cdot(\psi_1-\psi_2),\\ q_2&=\Delta\psi_2-F\cdot(\psi_2-\psi_1). \end{aligned} \tag{1.2}$$

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Here $(x,y)\in O:=(0,L)\times(0,L)\in\mathbb{R}^2$, L is the characteristic scale for horizontal length of the flows; F is positive defined by (see also [2], p.87)

$$F=\frac{f_0^2}{gh}\frac{\rho_0}{\rho_2-\rho_1}, \tag{1.3}$$

g is the gravitational acceleration, h is the depth of layers with the assumption that the depth of top and bottom layers is equal, ρ_1 and ρ_2 are the densities ($\rho_2>\rho_1$) of top and bottom layers, respectively; ρ_0 is the characteristic scale for density of the flows, $f_0+\beta y$ (with f_0,β constants) is the Coriolis parameter and β is the meridional gradient of the Coriolis parameter; $\nu>0$ is the viscosity. Note that $r=f_0\delta_E/(4h)$ is the Ekman constant ([3], p.29). Here $\delta_E=\sqrt{2\nu/f_0}$ is the Ekman layer thickness ([1], p.188). Moreover, $J(h,g)=h_xg_y-h_yg_x$ is the Jacobi operator and $\Delta=\partial_{xx}+\partial_{yy}$ is the Laplace operator in $\mathbb{R}^2$. Finally, $f(x,y)$ is the mean (deterministic) wind forcing, two-sided Wiener processes $W_1(t)$ and $W_2(t)$, which describe the fluctuating part of the external wind forcing in the fluid, either are mutual independent or $W_1(t)=W_2(t)$. In this paper, we consider the case when the covariance operators Q_1 and Q_2 of the Wiener processes $W_1(t)$ and $W_2(t)$ have a finite trace, respectively.

The two-layer quasigeostrophic flow model has been used as a theoretical and numerical model to understand basic mechanisms in large scale geophysical flows, such as baroclinic effects [1], wind-driven circulation [4, 5], the Gulf Stream [6], fluid stability [7] and subtropical gyres [3, 8]. Recently Salmon [9] introduced a generalized two-layer ocean flow model.

We assume Dirichlet boundary conditions for $\psi=(\psi_1,\psi_2)$:

$$\psi|_{\partial O}=\Delta\psi|_{\partial O}=0. \tag{1.4}$$

We also assume an appropriate initial condition $\psi(x,y,0)=\psi_0(x,y)$.

The stochastically forced quasigeostrophic model has been used to investigate various phenomena in geophysical flows [10–15]. This stochastic model has also be investigated in the context of stochastic dynamical systems [16–20].

For this stochastic two-layer model, following [16], we can establish the well-posedness and the existence of pullback attractors. The purpose of this paper is to establish the synchronization for the stochastic two-layer model.

In this paper, we first recall some basic facts about random dynamical systems in Section 2. In Section 3, we establish the well-posedness of the stochastic two-layer quasigeostrophic model by transforming it into a coupled system of random partial differential equations. In Section 4, we show the existence of pullback attractors for the random two-layer model. In Section 5, we establish the synchronization for the random two-layer model. In Section 6, the main results concerning stochastic two-layer quasigeostrophic model are established.