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EXPLICIT-IMPLICIT SPLITTING SCHEMES FOR SOME SYSTEMS OF EVOLUTIONARY EQUATIONS

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This paper is dedicated to the 65-th birthday of Professor Francisco J. Lisbona

Abstract. In many applied problems, the individual components of the unknown vector are interconnected and therefore splitting schemes are applied in order to get a simple problem for evaluating unknowns at a new time level. On the basis of additive schemes (splitting schemes), there are constructed efficient computational algorithms for numerical solving the initial value problems for systems of time-dependent PDEs. The present paper deals with computational algorithms that are based on using explicit-implicit approximations in time. Typically, additive operator-difference schemes for systems of evolutionary equations are constructed for operators that are coupled in space. Here we investigate more general problems, where we have coupling of derivatives in time for components of the solution vector.

Key words. Evolutionary problem, splitting scheme, stability of operator-difference schemes, additive operator-difference schemes.

1. Introduction

In solving applied problems, we deal with boundary value problems for systems of time-dependent PDEs. To construct computational algorithms for such problems, we approximate the equations taking into account appropriate initial and boundary conditions. Approximation in space is based on finite difference schemes, finite element procedures or finite volume methods [8, 12, 19, 20]. Special requirements are applied to the approximation in time for numerical solving problems for systems of equations [1, 9, 13]. In addition to general requirements to satisfy the conditions of approximation and stability, it is necessary to keep in mind the issues of computational implementation of the constructed schemes, i.e., the issue how to solve of the corresponding discrete problem at a new time level. In this regard, the most impressive results are associated with the construction of special additive operator-difference schemes (splitting schemes) [15, 24].

Additive schemes (operator-splitting schemes) are used to solve various unsteady problems [15, 20, 24, 30]. They are designed for the efficient computational implementation of the corresponding discrete problem defining the approximate solution at a new time level. The transition to a chain of simpler problems allows us to construct efficient difference schemes. We speak of splitting with respect to spatial variables (locally one-dimensional schemes). In some cases, it is useful to separate subproblems of distinct nature — we have splitting into physical processes. Regionally additive schemes (domain decomposition methods) are focused on constructing computational algorithms for parallel computers.

The main theoretical results on stability and convergence of additive schemes have been obtained for scalar evolutionary first-order equations and, in some cases,

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for second-order equations [15, 20, 24, 30]. In computational practice, it is essential to construct splitting schemes for systems of evolutionary equations. For example, vector problems have individual components of the unknown vector that are interconnected with each other. In this case, the use of appropriate splitting schemes is intended to obtaining simple problems for the individual components of the solution at a new time level.

For standard parabolic and hyperbolic systems of equations with a self-adjoint elliptic operator, additive schemes have been constructed in [20] using the regularization principle for difference schemes. Splitting schemes for systems of equations can be constructed employing the triangular splitting of a problem operator into the sum of operators adjoint to each other, i.e., using the alternating triangle method developed by Samarskii. Additive schemes of this type were used in [14] for dynamic problems of elasticity. A similar approach [25, 29] was applied to problems of an incompressible fluid with a variable viscosity. Additive schemes for transient problems of electrodynamics were considered in [27].

The above-mentioned classes of additive operator-difference schemes for evolutionary equations are based on an additive splitting of the leading operator into several terms. For many problems of practical interest, it is interesting to investigate the problems that have an additive representation for the operator at the time derivative. In the first publication on this subject [28], there were proposed and examined vector additive operator-difference schemes, where the operator at the time derivative was split into the sum of self-adjoint and positive definite operators.

Among additive schemes, we highlight explicit-implicit schemes, where the different nature of terms of the problem operator is taken into account via inhomogeneous approximations in time. Explicit-implicit schemes are widely used for the numerical solution of convection-diffusion problems. Various variants of inhomogeneous discretization in time are given in [2]. One or another explicit approximation is applied to the convective transport operator, whereas the diffusive transport operator is approximated implicitly. Thus, the most severe restrictions on a time step due to diffusion are removed. In view of the subordination of the convective transport operator to the diffusive transport operator, we have already proved unconditional stability of the above-considered explicit-implicit schemes for time-dependent convection-diffusion problems. Similar techniques are used in the analysis of diffusion-reaction problems. In this case (see, e.g., [17]), the diffusive transport is treated implicitly, whereas for reactions (source terms), explicit approximations are used. Such explicit approximations demonstrate obvious advantages for problems with nonlinear terms describing reaction processes. Detailed consideration of the implicit-explicit (IMEX) algorithms is given in the book [11] containing references to other works in this field of research.

In this paper, we propose splitting schemes for additive representation of the leading operator of the problem, i.e., the operator at the time derivative. We separate the diagonal part of a problem operator matrix and employ explicit-implicit approximations in time. The paper is organized as follows. In Section 1, we formulate the initial-boundary value problem for a system of PDEs. After some discretization in time, we obtain the Cauchy problem for a system of evolutionary equations. The standard two-level operator-difference scheme is discussed in Section 2. Section 3 deals with the construction of the explicit-implicit scheme by means of separating the diagonal part of the leading operator of the problem. The general problem of splitting the operator at the time derivative is discussed in Section 4.