J. Partial Diff. Eqs. **20**(2007), 252–264

Vol.20 No.3

ORBITAL STABILITY OF SOLITARY WAVES FOR GENERALIZED ZAKHAROV SYSTEM*

Yang Hui
(School of Mathematics Yunnan Normal University, Kunming 650092;
The Graduate School of China Academy of Engineering Physics,
P.O. Box 2101, Beijing 100088, China)
(E-mail: yh_m1026@yahoo.com.cn)
(Received Dec. 18, 2006)

Abstract This paper considers the stability of the solitary waves for the generalized Zakharov system. By applying the abstract theory of Grillakis M. *et al.* and detailed spectral analysis, we obtain the stability of the solitary waves.
Key Words Solitary waves; orbital stability; generalized Zakharov system.
2000 MR Subject Classification 35Q55, 35B35.
Chinese Library Classification O175.21.

1. Introduction

In the interaction of laser-plasma the system of Zakharov equation plays an important role [1, 2]. This system attracted many scientists' wide interest and attention. The formation, evolution and interaction of the Langmuir solutions differ from the solutions of the KdV equation.

In this paper we consider the following generalized Zakharov system

$$\begin{cases} i\varepsilon_t + \varepsilon_{xx} - n\varepsilon + \alpha|\varepsilon|^p\varepsilon + \beta|\varepsilon|^q\varepsilon = 0 \\ n_{tt} - n_{xx} = (|\varepsilon|^2)_{xx} \end{cases} \quad x \in \mathbb{R}, \tag{1.1}$$

where the real unknown function $n(x,t)$ is the fluctuation in the ion density about its equilibrium value, and the complex unknown function $\varepsilon(x,t)$ is the slowly varying envelope of highly oscillatory electron field. α, β, $p > 0$, and $q > 0$ are constants.

For the case $\alpha = 0$ and $\beta = 0$ the equation (1.1) becomes the classical Zakharov system. The global existence of solutions for the initial value problem of (1.1) is investigated in [3] by Galerkin Method. And in [4] by applying the abstract theory of Grillakis M. *et al.* [5, 6], the orbital stability of solitary waves is obtained.

*The research is supported by the Scientific Research Foundation of Yunnan Provincial Department and the Natural Science Foundation of Yunnan Province(2005A0026M)

For real constants α, β and $0 < p, q$, by Galerkin method the local and global existence of initial value problem for equations (1.1) can be obtained.

Theorem 1 *Suppose* $n(x,0) \in H^6$, $n_t(x,0) \in H^4$, $\varepsilon(x,0) \in H^8$ *and all of them with periodic* 2π, *and when the problem* (1.1) *satisfies one of the following conditions*

(1) *if* $\alpha \leq 0$, $\beta \leq 0$, *for all* $p, q > 0$,

(2) *if* $\alpha \leq 0, \beta > 0$, *for* $p > 0$ *and* $0 < q < 4$,

(3) *if* $\alpha > 0, \beta > 0$ *for* $0 < p, q < 4$,

(4) *if* $\alpha > 0, \beta \leq 0$, *for* $q > 0$ *and* $0 < p < 4$,

the global classical solutions of initial value problem (1.1) *exist.*

In this article, we use the same method as in [4] to consider the orbital stability of the solitary waves of Eq.(1.1) for the case $p = 2$ and $q = 4$.

2. The Existence of Solitary Waves

Consider the following generalized Zakharov system

$$\begin{cases} i\varepsilon_t + \varepsilon_{xx} - n\varepsilon + \alpha|\varepsilon|^2\varepsilon + \beta|\varepsilon|^4\varepsilon = 0 \\ n_{tt} - n_{xx} = (|\varepsilon|^2)_{xx} \end{cases} \quad x \in \mathbb{R}. \tag{2.1}$$

Let

$$\varepsilon(x,t) = e^{-i\omega t} e^{iq(x-vt)} a(x - vt), \tag{2.2}$$

$$n(x,t) = n(x - vt) \tag{2.3}$$

be the solitary waves of Eq.(2.1), where ω, q, v are real numbers, $a(x-vt)$ and $n(x-vt)$ are real functions. Denote $\xi = x - vt$.

Put Eqs. (2.2), (2.3) into Eq. (2.1) then we obtain

$$(v^2 - 1)n''(\xi) = (a^2(\xi))'', \tag{2.4}$$

$$a'' + i(2q - v)a' + (\omega + vq - q^2)a + \frac{1}{1 - v^2}a^3 + \alpha a^3 + \beta a^5 = 0. \tag{2.5}$$

By Eq.(2.4), we have

$$n(\xi) = \frac{-a^2(\xi)}{1 - v^2}. \tag{2.6}$$

Eq. (2.5) implies

$$2q = v, \tag{2.7}$$

$$a'' + (\omega + vq - q^2)a + (\alpha + \frac{1}{1 - v^2})a^3 + \beta a^5 = 0. \tag{2.8}$$

Let $a^2(\xi) = \varphi(\xi)$, we have

$$\frac{1}{2}\varphi''\varphi^{-\frac{1}{2}} - \frac{1}{4}\varphi^{-\frac{3}{2}}(\varphi')^2 + l\varphi^{\frac{1}{2}} + m\varphi^{\frac{3}{2}} + \beta\varphi^{\frac{5}{2}} = 0, \tag{2.9}$$