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A NOTE ON "SMALL AMPLITUDE SOLUTIONS OF THE GENERALIZED IMBQ EQUATION"

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Abstract Global existence of small amplitude solution and nonlinear scattering result for the Cauchy problem of the generalized IMBq equation were considered in the paper titled "Small amplitude solutions of the generalized IMBq equation" [1]. It is a pity that the authors overlooked the bad behavior of low frequency part of $S(t)\psi$ which causes troubles in L^∞ and H^s estimates. In this note, we will present a new proof of global existence under same conditions as in [1] but for space dimension $n \geq 3$.

Key Words IMBq equation; Duhamel's principle; Hölder inequality; Gronwall inequality; Hausdorff-Young inequality.

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1. Introduction

In the paper titled "Small amplitude solutions of the generalized IMBq equation" [1], global existence of small amplitude solution and scattering theory for the Cauchy problem of the generalized IMBq equation

$$\begin{cases} u_{tt} - \Delta u_{tt} - \Delta u = \Delta f(u), \\ u(0) = \varphi(x), \quad u_t(0) = \psi(x) \end{cases} \tag{1.1}$$

were considered, where $u(x,t)$ denotes the unknown function, $f(u)$ is the given nonlinear function, $\varphi(x)$ and $\psi(x)$ are the given initial data, subscript t indicates the partial derivative with respect to t, Δ denotes the Laplace operator in $\mathbb{R}^n$. For the physic background and researches of the IMBq equation, one may refer the readers to [1-4] and the references therein.

Through combining the factors with functions respectively, the authors used the special case of Lemma 2.1 with $F \equiv 1$ to deal with three terms of the right hand side of (2.2) below to get the estimates for linearized equation. Together with the estimates for the nonlinearity $f(u)$, global existence of small amplitude solution for (1.1) followed by the usage of Banach fixed point Theorem.

It is a pity that the authors overlooked the infinity of the factor $\frac{\sqrt{1+|\xi|^2}}{|\xi|}(\xi \to 0)$ in the dealing with the low frequency part of $S(t)\psi$, using

$$\|(I-\Delta)^{\frac{1}{2}}(-\Delta)^{-\frac{1}{2}}\psi\|_{L^1} + \|(I-\Delta)^{\frac{1}{2}}(-\Delta)^{-\frac{1}{2}}\psi\|_{H^s} \le C(\|\psi\|_{L^1} + \|\psi\|_{H^s}) \tag{1.2}$$

which is not correct to prove Lemma 2.4 of [1].

In this paper, we deduce Lemma 2.1 for general $F(\xi) = F(|\xi|) = F(r)$ instead of $F \equiv 1$ on the basis of Van der Corput Lemma. Note that $\frac{\sqrt{1+|\xi|^2}}{|\xi|}$ is L^1 and L^2 in multi-dimensional case. Through different choices of $F(\xi)$, we deal with three terms in the RHS of (2.2) differently to get the estimates for linearized equation. Together with the estimates obtained in [1] for $f(u)$, we finally get the result of the present paper without any other additional condition on initial data.

2. Some Estimates

Firstly, let's consider the linearized equation

$$\begin{cases} u_{tt} - \Delta u_{tt} - \Delta u = \Delta g(x,t), \\ u(0) = \varphi(x), \quad u_t(0) = \psi(x) \end{cases} \tag{2.1}$$

of (1.1). By Duhamel's principle, the solution of (2.1) can be written as

$$u(x,t) = \partial_t S(t)\varphi(x) + S(t)\psi(x) + \int_0^t T(t-\tau)g(x,\tau)\mathrm{d}\tau \tag{2.2}$$

where $T(t) = S(t)(I-\Delta)^{-1}\Delta$, and

$$\partial_t S(t)\varphi(x) = (2\pi)^{-n}\int_{\mathbb{R}^n} e^{ix\cdot\xi}\cos\frac{t|\xi|}{\sqrt{1+|\xi|^2}}\hat{\varphi}(\xi)\mathrm{d}\xi, \tag{2.3}$$

$$S(t)\psi(x) = (2\pi)^{-n}\int_{\mathbb{R}^n} e^{ix\cdot\xi}\frac{\sqrt{1+|\xi|^2}}{|\xi|}\sin\frac{t|\xi|}{\sqrt{1+|\xi|^2}}\hat{\psi}(\xi)\mathrm{d}\xi, \tag{2.4}$$

$$\begin{aligned}&\int_0^t T(t-\tau)g(x,\tau)\mathrm{d}\tau \\ &\quad = -(2\pi)^{-n}\int_0^t\left[\int_{\mathbb{R}^n} e^{ix\cdot\xi}\frac{|\xi|}{\sqrt{1+|\xi|^2}}\sin\frac{(t-\tau)|\xi|}{\sqrt{1+|\xi|^2}}\hat{g}(\xi,\tau)\mathrm{d}\xi\right]\mathrm{d}\tau\end{aligned} \tag{2.5}$$

with $\hat{f}(\xi) = \int_{\mathbb{R}^n} e^{-ix\cdot\xi}f(x)\mathrm{d}x$.

As in [4], in order to estimate $u(x,t)$, we still need the following lemma derived on the basis of Van der Corput lemma [5,6] to deal with the medium parts of all three terms in the right hand side of (2.2).