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NEW EXPLICIT SOLUTIONS TO THE (2+1)-DIMENSIONAL BROER-KAUP EQUATIONS

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Abstract Applying the homogeneous balance method, we have found the explicit and soliton solutions and given a successive formula of finding explicit solutions to the (2+1)-dimensional Broer-Kaup equations. Moreover, by using the Lie group method, we have discussed the similarity solutions to the (2+1)-dimensional Broer-Kaup equations.
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1. Introduction

For the (1+1)-dimensional Broer–Kaup (BK) system

$$H_t = (H^2 + 2G - H_x)_x, \tag{1}$$
$$G_t = (G_x + 2GH)_x, \tag{2}$$

Matsukidaira and Satsuma obtained their expressions of trilinear forms [1]. Kupershmidt proved the BK system has an infinite number of conserved densities and higher commuting flows [2]. Hirota obtained the N-soliton solutions of the BK system [3,4]. This system is a generalization of the classical dispersiveless long wave equations [2]. Using the Darboux transformation related symmetry constraints of the KP (Kadomtsev-Petviashvili) equation, Lou and Hu have obtained the following Broer-Kaup (BK) equations [5]

$$H_t = H_{xx} - 2HH_x - 2\partial_y^{-1}G_{xx}, \tag{3}$$
$$G_t = -G_{xx} - 2\left(HG\right)_x, \tag{4}$$

where $\partial_y^{-1} f = \int f dy$. Obviously, when $y = x$ and t is replaced by $-t$, the BK equations (3) and (4) can be changed into (1) and (2) respectively. Therefore, the BK equations (3) and (4) are also the generalization of (1) and (2) in (2+1)-dimensions space-time {x,y,t}.

In this paper we consider the following (2+1)-dimensional BK equations

$$H_{ty} = H_{xxy} - 2(HH_x)_y - 2G_{xx}, \tag{5}$$

$$G_t = -G_{xx} - 2\left(HG\right)_x, \tag{6}$$

which is equivalent to (3) and (4). In [6], Ruan and Chen discussed the Plainleve property,symmetries to the BK equations (5) and (6). Our aim is to find the explicit solutions of the BK equations (5) and (6) . In Section 2, by applying the extended homogeneous balance method [7], we first reduce the BK equations (5) and (6) to two linear equations with a constraint condition. In Section 3, based on the reduced equation, we obtain some new explicit solutions to the BK equations (5) and (6). In Section 4, we discuss the optimal system of one- dimensional subalgebras corresponding to the reduced equation. In Section 5, we find some similarity solutions to the reduced equation and can get some new solutions to the BK equations (5) and (6). We generalize the results in [8].

2. Reduction of BK Equations

In this section , we shall get a reduction of the equations (5) and (6) by applying the extended homogeneous balance method . To balance the nonlinear term $(HH_x)_y$ (or $(HG)_x$) and the third order derivative term H_{xxy} (or second order derivative term G_{xx}) in the equation (5) (or (6)), we take the following form solutions of the BK equations

$$H = P\left(x, y, t\right) + h^{'}\left(\phi\right)\phi_x, \tag{7}$$

$$G = Q\left(x, y, t\right) + g^{''}\left(\phi\right)\phi_x\phi_y + g^{'}\phi_{xy}, \tag{8}$$

where the functions $h\left(\phi\right), g\left(\phi\right)$ and $\phi\left(x, y, t\right)$are to be determined, P and Q are the given solutions of the equations (5) and (6), $h^{'}\left(\phi\right) = \partial h/\partial\phi$ and $g^{''}\left(\phi\right) = \partial^2 g/\partial^2\phi$. Substituting (7) , (8) into (5) , (6) and using Maple , we can find that $h\left(\phi\right) = g\left(\phi\right) = \ln\left(\phi\right)$ and yields

$$A = h^{(3)}R\phi_x\phi_y + h^{(2)}\left[R\phi_{xy} + R_x\phi_y + R_y\phi_x\right] + h^{'}R_{xy} + P_1, \tag{9}$$

$$\begin{aligned} B = & h^{(3)}R\phi_x\phi_y + h^{(2)}\left[R\phi_{xy} + R_x\phi_y + R_y\phi_x\right] + h^{'}R_{xy} \\ & + 2h^{'}\left[\left(Q - P_y\right)\phi_x\right]_x + Q_1, \end{aligned} \tag{10}$$

where

$$\begin{aligned} A &:= H_{ty} - H_{xxy} + 2\left(HH_x\right)_y + 2G_{xx}, \\ B &:= G_t + G_{xx} + 2\left(HG\right)_x, \\ R &:= \phi_t + \phi_{xx} + 2P\phi_x, \\ P_1 &:= P_{ty} - P_{xxy} + 2\left(PP_x\right)_y + 2Q_{xx}, \\ Q_1 &:= Q_t + Q_{xx} + 2\left(QP\right)_x. \end{aligned}$$