J. Partial Diff. Eqs. **16**(2003), 97–110
Vol.16 No.2

EXPONENTIAL ATTRACTOR FOR COMPLEX GINZBURG-LANDAU EQUATION IN THREE-DIMENSIONS

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(Received Jan. 14, 2002; revised Sep. 16, 2002)

Abstract In this paper, we consider the complex Ginzburg-Landau equation (CGL) in three spatial dimensions

$$u_t = \rho u + (1 + i\gamma)\Delta u - (1 + i\mu)\,|u|^{2\sigma}\,u\ , \tag{1}$$

$$u(0, x) = u_0(x), \tag{2}$$

where u is an unknown complex-value function defined in 3+1 dimensional space-time R^{3+1}, Δ is a Laplacian in R^3, $\rho > 0$, γ, μ are real parameters. $\Omega \in R^3$ is a bounded domain. We show that the semigroup $S(t)$ associated with the problem (1), (2) satisfies Lipschitz continuity and the squeezing property for the initial-value problem (1), (2) with the initial-value condition belonging to $H^2(\Omega)$, therefore we obtain the existence of exponential attractor.

Key Words Ginzburg-Landau equation; exponential attractor; squeezing property.

2000 MR Subject Classification 35B30, 35B45, 35K55.

Chinese Library Classification O175.29.

1. Introduction

The natural object which is used to describe the behavior of the solution of nonlinear evolutionary PDE's is the global attractor of these PDE's. This is the set in the phase space to which all trajectories beginning in bounded sets ultimately converge. The existence of global attractors and estimates of their fractal and Hausdorff dimension were obtained for many equations of mathematical physics. Now, other objects have invented to describe large-time dynamics of PDE's: exponential attractor(see Eden et al.(1993)[1] and Eden et al.(1994)[2]). The exponential attractors have finite fractal dimension and contain global attractor, and the rate of attraction to all trajectories is uniformly exponential.

The generalized complex Ginzburg-Landau (CGL) equation describes the evolution of a complex-valued $u = u(x,t)$ by

$$u_t = \rho u + (1 + i\gamma)\Delta u - (1 + i\mu)|u|^{2\sigma}u \ ,$$

It has a long history in physics as a generic amplitude equation near the onset of instabilities that lead to chaotic dynamics in fluid mechanical systems, as well as in the theory of phase transitions and superconductivity. It is a particularly interesting model because it is a dissipative version of the nonlinear Schrödinger equation,A Hamiltonian equation which can possess solutions that form localized singularities in finite time. Ghidaglia and Héorn [3], Doering et al [4] studied the finite dimensional Global attractor and related dynamic issues for the one or two spatial dimensional GLE with cubic nonlinearity ($\sigma = 1$).Bartuccelli, Constantin, Doering, Gibbon and Gisselfalt [5] deal with the "soft" and "hard" turbulent behavior for this equation. Doering, Gibbon and Levermore [6] present the existence and uniqueness of global weak solution and strong solution for the equation (1.1) in all spatial dimensions and for all degree of nonlinearity $\sigma > 0$. Mielke [7] discussed the solution of (1.1) in weighted L^p space and derived some new bounds and investigated some properties of attractors. B.Guo and B.Wang study the exponential attractor for the derivative Ginzburg-Laudan equation in two-dimensional space.

In this paper, we consider the complex Ginzburg-Landau equation (CGL) in three spatial dimensions

$$u_t = \rho u + (1 + i\gamma)\Delta u - (1 + i\mu)|u|^{2\sigma}u \ , \tag{1.1}$$

$$u(0,x) = u_0(x), \quad x \in \Omega, \tag{1.2}$$

where $\Omega = (0, L_1) \times (0, L_2) \times (0, L_3), u$ is Ω-periodic. Our aim is to show that the semigroup $S(t)$ associated with the problem (1.1), (1.2) existence of the exponential attractor.

The rest of this paper is arranged as follows. First, in Section 2 we present the results related to this system and give the construction of some compact positively invariant sets. Next, in Section 3 we show the Lipschitz continuity and squeezing property of the dynamic system $S(t)$ for every $T > 0$. Finally, we obtain our main result, exponential attractor for the problem(1.1), (1.2) in three dimensions.

2. Main Result

Throughout this paper, let $H = L^2_{per}(\Omega)$, $H^k = H^k_{per}(\Omega)$ denote the usual Sobolev space. And $\|\cdot\|$ denote the $L^2(\Omega)-$norm. Here and after c denotes any constant.