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# GLOBAL CLASSICAL SOLUTIONS FOR ONE DIMENSIONAL HYDROMAGNETIC FLOW WITH DISSIPATIVE TERMS*

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**Abstract** This paper concerned with the classical solutions to system of one dimensional hydromagnetic dynamics with dissipative mechanism. Under certain hypotheses on the initial data, the global existence and the formation of singularities for classical solution are obtained. Our results show that the damping dissipation is strong enough to preserve the smoothness of the classical solution.

**Key Words** Hydromagnetic flow; Cauchy problem; Classical solution; Dissipative mechanism; Singularity.

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## 1. Introduction

In this paper we are interested in the following one dimensional hydromagnetic flow

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with being non-strictly hyperbolic system in Lagrangian representation ([1]):

$$\begin{cases} \dfrac{\partial v}{\partial t} - \dfrac{\partial u}{\partial x} = 0, \\ \dfrac{\partial u}{\partial t} + \dfrac{\partial P(v,s)}{\partial x} + \dfrac{1}{4\pi}\left(H_y \dfrac{\partial H_y}{\partial x} + H_z \dfrac{\partial H_z}{\partial x}\right) + 2\alpha u = 0, \\ \dfrac{\partial H_y}{\partial t} + \dfrac{H_y}{v}\dfrac{\partial u}{\partial x} = 0, \\ \dfrac{\partial H_z}{\partial t} + \dfrac{H_z}{v}\dfrac{\partial u}{\partial x} = 0, \\ \dfrac{\partial s}{\partial t} = 0, \end{cases} \tag{1.1}$$

where $\alpha > 0$ is constant, $u, v, H_y, H_z$ and $s$ are unknown and denote the velocity, the specific, the $y$ and $z$ directional components of magnetic field $\vec{H}$ (here we assume that the $x$ directional component $H_x$ is constant, and may suppose that $H_x \equiv 0$), and the entropy respectively, the pressure $P = P(v, s)$ satisfies:

$$\frac{\partial P}{\partial v} < 0, \quad \frac{\partial^2 P}{\partial v^2} > 0, \quad \forall v > 0. \tag{1.2}$$

For $\alpha = 0$, the formation of singularities in magnetohydrodynamics waves with the "large" initial datum which has compact support, periodicity or certain decay properities as $|x| \to \infty$ (cf [2,3]), is exhausively studied. However,as to the case that $\alpha > 0$, the global existence and the blowup phenomena of classical solution are hardly studied.

In this paper,we shall investigate the global smooth resolvability for Cauchy problem for the system (1.1), and an outline follows: the main results are stated in Section 2 (Theorem 1, Theorem 2, Theorem 3), Section 3 presents some prelimaries, the proof of global existence theorems can be seen in Section 4, and Section 5 gives the proof of Theorem 3.

## 2. Main Results

Let

$$H_1 = vH_y, \quad H_2 = vH_z, \tag{2.1}$$