J.Partial Diff. Eqs. **12**(1999), 229–242
©International Academic Publishers Vol.12, No.3

ON THE EXISTENCE OF THE GLOBAL SMOOTH PROCESS FOR ONE-DIMENSIONAL NONLINEAR THERMOVISCOELASTIC MATERIALS WITH FIXED AND THERMALLY INSULATED ENDPOINTS

Li Hailiang and Pan Ronghua
(Institute of Mathematics, Academia Sinica, Beijing 100080, China)
(Received Aug. 26, 1997)

Abstract The system of balance laws of mass, momentum and energy for one-dimensional nonlinear thermoviscoelastic material with fixed and thermally insulated endpoints is considered, and the problem of whether there exists a globally defined smooth thermoviscoelastic process is solved, i.e., there exists such a process.
Key Words Global smooth solution; nonlinear thermoviscoelastic materials.
Classification 35M10, 73C35, 73B30.

1. Introduction

In this paper we will consider the existence of the global smooth process to one-dimensional nonlinear thermoviscoelastic materials with fixed and thermally insulated endpoints. This is an open problem proposed by Dafermos in [1].

The Lagrangian referential form of conservation laws of mass, momentum, and energy for one-dimensional materials with the referencial density $\rho_0 = 1$ is

$$\begin{aligned} &u_t - v_x = 0 \\ &v_t - \sigma_x = 0 \\ &\left[e + \frac{v^2}{2}\right]_t - [\sigma v]_x + q_x = 0 \end{aligned} \tag{1.1}$$

and the second law of thermodynamics is expressed by the Clausius-Duhem inequality

$$\eta_t + \left(\frac{q}{\theta}\right)_x \geq 0 \tag{1.2}$$

where $u, v, \sigma, e, \eta, \theta$ and q denote orderly deformation gradient, velocity, stress, internal energy, special entropy, temperature and heat flux. u, e and θ may take positive values.

For one-dimensional, homogeneous, thermoviscoelastic materials, internal energy, stress, entropy and heat flux are given by constitutive relations

$$e = \hat{e}(u,\theta), \sigma = \hat{\sigma}(u,\theta,v_x), \eta = \hat{\eta}(u,\theta), q = \hat{q}(u,\theta,\theta_x) \tag{1.3}$$

which, according to (1.2), must satisfy

$$\begin{aligned}&\hat{\sigma}(u,\theta,0)=\hat{\psi}_u(u,\theta),\quad \hat{\eta}(u,\theta)=-\hat{\psi}_\theta(u,\theta)\\&[\hat{\sigma}(u,\theta,w)-\hat{\theta}(u,\theta,0)]w\ge 0,\quad \hat{q}((u,\theta,g)g\le 0\end{aligned}\tag{1.4}$$

where $\hat{\psi}=\hat{e}-\theta\hat{\eta}$ is the Helmholtz free energy.

We consider here a body with reference configuration the interval [0, 1] with fixed endpoints and thermal insulation, i.e.,

$$\begin{aligned}v(0,t)=v(1,t)=0,\quad t\ge 0\\q(0,t)=q(1,t)=0,\quad t\ge 0\end{aligned}\tag{1.5}$$

The initial data of deformation gradient, velocity and temperature are expressed by

$$u(x,0)=u_0(x)>0,\ v(x,0)=v_0(x),\ \theta(x,0)=\theta_0(x)>0\tag{1.6}$$

In 1982, Dafermos [1] considered the system (1.1) with the following boundary conditions of stress free and thermal insulation:

$$\begin{aligned}q(0,t)=q(1,t)=0,\quad t\ge 0\\\sigma(0,t)=\sigma(0,t)=0,\quad t\ge 0\end{aligned}\tag{1.7}$$

Using Leray-Schauder fixed point theorem, he got the global smooth process to the problem (1.1), (1.6)–(1.7). Recently, mainly based on the techniques in Dafermos and Hsiao [2] and Dafermos [1], Jiang [3] established the smooth solution for (1.1) with dash pot or stress free and constant temperature at endpoints. The initial value problems are considered by Zheng and Shen [4] and Kim [5]. As for the large time behavior of classical solutions to (1.1) is concerned, the nonlinear phenomena of phase transition of solutions was found by Hsiao and Luo [6], and by Hsiao and Jian [7], which is an extension of the results obtained by Andrews and Ball [8]. When the material is gas, similar study was made, such as [9–15] and references cited there.

Here, we will show the global existence of smooth solutions to (1.1), (1.5)–(1.6) for the same solid-like material as that in Dafermos [1]. The first step to establish the global classical solution is to get the *a priori* bound of the deformation gradient u, if we have no restriction on the behaviors of $\hat{e}(u,\theta)$, $\hat{p}(u,\theta)$, $\hat{k}(u,\theta)$ at $u=0_+$ and $u=+\infty$. This can be guaranteed in Dafermos [1] by the property of $p(u,\theta)$, i.e. (1.11), and in Jiang [3] with other monotone request on $p(u,\theta)$ with respect to the deformation gradient u. In our case, however, the techniques used in [1], [3] do not work, which means that further study on the property of the system is needed.

Following Dafermos [1], we consider the global existence problem of solution to (1.1), (1.5)–(1.6) for linearly viscous material

$$e=\hat{e}(u,\theta),\ \sigma=-\hat{p}(u,\theta)+\hat{\mu}(u)v_x,\ q=-\hat{k}(u,\theta)\theta_x\tag{1.8}$$