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ON THE EXISTENCE OF GLOBAL OSCILLATION WAVES FOR A CLASS OF 3 × 3 SEMILINEAR HYPERBOLIC EQUATIONS

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Abstract In this paper, we prove the global existence of oscillation waves for a class of 3 × 3 semilinear hyperbolic equations by applying the Young measures and two-scale Young measures which are associated with the solution sequence of the system.

Key Words Young measures; geometric optics; 3-web curvatures.
Classification 35C20, 35L60.

1. Introduction

Recently, rapid progress has been made on the rigorous justification of weakly non-linear geometric optics [1-6]. In [1], J.L. Joly et al proved that: if the matrics $A(t,x) = \text{diag}\,(\lambda_1(t,x), \lambda_2(t,x), \cdots, \lambda_N(t,x))$ with $\lambda_1(t,x) < \cdots < \lambda_N(t,x)$, for $(t,x) \in \mathcal{R}^+ \times \mathcal{R}$, the Cauchy problem of the following system:

$$\begin{cases} (\partial_t + A(t,x)\partial_x)u^\varepsilon = f(t,x,u^\varepsilon) \\ u_k^\varepsilon(t,x)\mid_{t=0} = H_k\Big(x, \dfrac{\varphi_k(0,x)}{\varepsilon}\Big) + o(1) \quad \text{in} \quad L^\infty[y_-, y_+]\ k = 1, \cdots, N \end{cases}$$

has a solution $u^\varepsilon(t,x)$, and some T independent of ε, such that the geometric approximation $u^\varepsilon(t,x) = \sum_{k=1}^{N} \mathcal{U}_k\Big(t, x, \dfrac{\varphi_k(t,x)}{\varepsilon}, \dfrac{1}{\varepsilon}\Big) + o(1)$ is valid in $L^\infty(\Omega_T)$, where $\Omega_T = \Omega_0 \cap \{t \le T\}$, $\Omega_0 = \{(t,x) \in \mathcal{R}^+ \times \mathcal{R} \mid 0 \le t \le T_1, \gamma_N(t,0,y_-) \le x \le \gamma_1(t,0,y_+)\}$, and $(t, \gamma_i(t,0,y))$ is the integral curve of $\partial_t + \lambda_i(t,x)\partial_x$ which issues from $(0,y)$, $\varphi_k(t,x)$ is the solution of the following eikonal equations

$$\begin{cases} (\partial_t + \lambda_k(t,x)\partial_x)\varphi_k(t,x) = 0 \\ \varphi_k(t,x)\mid_{t=0} = \varphi_k(0,x) \end{cases} \tag{1}$$

under the assumptions that the phase functions $\{\varphi_k(t,x)\}$ satisfy the closedness and transversality properties.

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In this paper, we will study the global existence of oscillation waves for the following 3×3 semilinear equations:

$$\begin{cases} X_i(t,x)u_i^\varepsilon(t,x) = f_i(t,x,u^\varepsilon), & (t,x) \in \mathcal{R}^+ \times \mathcal{R} \\ u_i^\varepsilon(t,x)\mid_{t=0} = \mathcal{U}_i^0\left(x, \dfrac{\varphi_i(x)}{\varepsilon}\right) \end{cases} \tag{2}$$

where $X_i(t,x) = \partial_t + \lambda_i(t,x)\partial_x$, $u^\varepsilon = (u_1^\varepsilon, u_2^\varepsilon, u_3^\varepsilon)$, $f(t,x,u^\varepsilon) = (f_1(t,x,u^\varepsilon), f_2(t,x,u^\varepsilon), f_3(t,x,u^\varepsilon))$ which is a continuous function from $[0,\infty) \times \mathcal{R} \times \mathcal{R}^3$, with $f(t,x,0) = 0$, and there is a constant K such that

$$\forall (t,x,u,v) \in \mathcal{R}^+ \times \mathcal{R} \times \mathcal{R}^3 \times \mathcal{R}^3, \quad |f(t,x,u) - f(t,x,v)| \le K|u - v| \tag{3}$$

$\lambda_i(t,x)$ and its first derivatives are uniformly bounded, $\mathcal{U}_i^0(x,\theta_i), \partial_{\theta_i}\mathcal{U}_i^0(x,\theta_i) \in L^2(\mathcal{R} \times \mathcal{T})$), and $\mathcal{T} = \dfrac{\mathcal{R}}{\mathcal{Z}}$. The difficulty lies in that: when the initial data $u_0^\varepsilon(x)$ are as those in (2), we can not apply the techniques of oscillatory integration which are used by J.L. Joly et al in [1]. But if the curvatures of 3-web of (X_1, X_2, X_3), $\mathcal{K}(t,x) \neq 0$, a.e. on $\mathcal{R}^+ \times \mathcal{R}$, we can apply the trilinear compensated compactness in [2], and Young measures as well as two-scale Young measures to overcome this difficulty. As for the definition of the curvatures of 3-web, please consult [2] for more details.

Theorem 1 *Let $\{X_i\}_{1\le i\le 3}$ be the three pairwise independent smooth vector fields such that the curvatures of the associated 3-web $\mathcal{K}(t,x) \neq 0$, a.e. on $\mathcal{R}^+ \times \mathcal{R}$, and arbitrary three phase functions $\varphi_i^0(x)$ with $(\varphi_i)'(x) \neq 0$ a.e. on $\mathcal{R}$. Then (2) has a unique solution $u^\varepsilon(t,x) \in C([0,\infty), L^2(\mathcal{R}))$, and*

$$u_i^\varepsilon(t,x) = \mathcal{U}_i\left(x, \frac{\varphi_i(t,x)}{\varepsilon}\right) + O(1) \text{ in } C([0,\infty), L^p(\mathcal{R})),\ 1 \le p < 2,\ i = 1,2,3 \tag{4}$$

where $\mathcal{U}_i(t,x,\theta)$ $(i = 1,2,3)$ are solutions of the following modulation equations:

$$\begin{cases} X_i\mathcal{U}_i(t,x,\theta_i) = Ef_i(t,x,\theta_i) \\ \mathcal{U}_i(t,x,\theta_i)\mid_{t=0} = \mathcal{U}_i^0(x,\theta_i), \quad i = 1,2,3 \end{cases} \tag{5}$$

with $Ef_1(t,x,\theta_1) = \int_0^1 f_1(t,x,\mathcal{U}_1(t,x,\theta_1), \mathcal{U}_2(t,x,\theta_2), \mathcal{U}_3(t,x,\theta_3))d\theta_2 d\theta_3$, similar definitions for $Ef_i(t,x,\theta_i)$, $i = 2,3$. Moecover, $\mathcal{U}_i(t,x,\theta_i), \partial_{\theta_i}\mathcal{U}_i(t,x,\theta_i) \in C([0,\infty), L^2(\mathcal{R}))$. And $\varphi_i(t,x)$, $i = 1,2,3$, are the solutions of the eikonal equations of (1).

Remark 1 . We may prove the corresponding Theorem 1 for special $N \times N$ systems, such as the example given in [2]. But, when the curvatures of 3-web of (X_1, X_2, X_3) vanish on a set which is not of null Lebesgue measure, we can not get the same conclusion by the methods given in this paper.

Remark 2 Note that our results globally hold on time, while in [1], the asymptotic expansions are valid only locally in time, moreover, our expansions haven't the scale, $\dfrac{1}{\varepsilon}$, and with no restriction on the phase functions $\varphi_i(t,x)$.