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LONG TIME BEHAVIOR OF NONLINEAR STRAIN WAVES IN ELASTIC WAVEGUIDES*

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Abstract The initial-boundary value problem of the propagation of nonlinear longitudinal elastic waves in an initially strained rod is considered. The rod is assumed to interact with the surrouding elastic and viscous external medium. The long time behavior of solutions is derived and global attractors in E_1 space is obtained.
Key Words Nonlinear strain waves; attractors.
Classification 35K57, 35B40.

1. Introduction

In some problems of nonlinear wave propagation in waveguides, the interaction of waveguides and the external medium and, therefore, the possibility of energy exchange through laternal surface of waveguide cannot be neglected, when the energy exchange between the rod and the medium is considered, for one case, there is a dissipation of a deformation wave in the viscous external medium, the general cubic double dispersion equation (CDDE) can be derived from Hamilton principle [1]

$$w_{tt} - w_{xx} = \frac{1}{4}(cw^3 + 6w^2 + aw_{tt} - bw_{xx} + dw_t)_{xx} \tag{1.1}$$

where d, c, a, b are some positive constants depending on the Young modulus E_0, the shear modulus μ, density of waveguide ρ and the Poisson coefficient ν, w is proportional to strain $\dfrac{\partial u}{\partial x}$, u is longitudinal displacement. The Equation (1.1) was studied in some literatures, the travelling wave solutions, depending upon the phase variable $z = x \pm vt$ were studied by [1–2], when $c = d = 0$, the strain solutions of the equation (1.1) were observed by [3–5], in this paper we will consider the global existence and uniqueness of solution of the equation (1.1) with initial-boundary value problem and prove the existence of global attractor. Since the nonlinear semigroup $S(t)$ generated by (1.1) is

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not compact in E_1, we cannot construct the global attractor by the method introduced by Temam [6] or Constantin, Foias and Temam [7], we here first apply the techniques developed by Ghidaglia [8] to show the existence of global weak attractor for (1.1) in E_1, then by energy equation and an idea of Ball [9] and [10] we conclude that the global weak attractor is actually the global strong attractor for $S(t)$ in E_1.

This paper is organized as follows. In Section 2, we derive uniform a priori estimates in time which enable us to show the existence of global solution. In Section 3, we first establish the existence of global weak attractor, then by decomposition of operator we prove that the global weak attractor is actually the global strong attractor in E_1.

2. The Solution Semigroup

Assume that length of elastic waveguides is 1, we consider the following cubic double dispersion equation

$$u_{tt} - u_{xx} = \frac{1}{4}(cu^3 + 6u^2 + au_{tt} - bu_{xx} + du_t)_{xx} \tag{2.1}$$

with the initial conditions

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x), \quad x \in (0,l) \tag{2.2}$$

and the boundary condition

$$u(0,t) = u(l,t) = 0, \quad u_{xx}(0,t) = u_{xx}(l,t) = 0 \tag{2.3}$$

We assume that

$$a, b, c, d \text{ are all positive constants} \tag{2.4}$$

and first establish some time-uniform *a priori* estimates on (u, u_t) in phase spaces $E_0 = H_0^1 \times L^2(I)$ and $E_1 = H^2 \cap H_0^1 \times H_0^1(I)$ respectively, where $I = (0,l)$.

Lemma 2.1 *Assume that* $u_0(x) \in H_0^1$, $u_1(x) \in L^2$ *then*

$$\|(u,u_t)\|_{E_0}^2 \le c(\|(u_0,u_1)\|_{E_0})e^{-\delta_0 t} + c \tag{2.5}$$

thus there exists $t_0 = t_0(R) > 0$ *such that*

$$\|(u,u_t)\|_{E_0} \le c_2, \quad t \ge t_0 \tag{2.6}$$

whenever $\|(u_0,u_1)\|_{E_0} \le R$, *where* $\|(u,u_t)\|_{E_0}^2 = \|u\|^2 + |u_t|_0^2$, $c(\|(u_0,u_1)\|_{E_0})$ *denote the constant depending only on norm of* u_0, u_1 *in* E_0 *space,* δ_0 *and* c *are independent of* u, $\|u\| = |u_x|_0 = \|u_x\|_{L^2}$.

Proof Let $u_t = \phi_{xx}$, from (2.1)-(2.3) we have

$$u_t = \phi_{xx}$$