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GLOBAL DYNAMICS OF A NONLINEAR BEAM EQUATION WITH STRONG STRUCTURAL DAMPING*

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Abstract In this paper, the global attractor, exponential attractor and flat inertial manifold are obtained for a nonlinear beam equation with strong structural damping.

Key Words Nonlinear beam equation; global attractor; exponential attractor; flat inertial manifold.
Classification 35K55, 35K57.

1. Introduction

In this paper we consider the following initial boundary value problem for a nonlinear heam equation, which is introduced in [1],

$$u_{tt}+\alpha\Delta^2 u+\delta\Delta^2 u_t-\Big\{a+b\int_\Omega|\nabla u|^2dx+q\Big[\int_\Omega(\nabla u\nabla u_t)dx\Big]^{2(m+\beta)+1}\Big\}\Delta u$$
$$=f,\quad x\in\Omega, t\geq 0 \tag{1}$$
$$u|_{\partial\Omega}=\Delta u|_{\partial\Omega}=0,\quad t\geq 0 \tag{2}$$
$$u(x,0)=u_0(x), u_t(x,0)=u_1(x),\quad x\in\Omega \tag{3}$$

where $u(x,t)$, $t\geq 0$, $x\in\Omega$, is the transverse deflection of the beam. Ω is a bounded open set in R^n. All the parameters α,δ,b and q are positive constants, but $a\in R^1$. The term $\delta\Delta^2u_t$ represents strong structural damping, $[a+b\|\nabla u\|^2]\Delta u$ is the tension of extensibility, and the last term on the left hand side is known as Balakrishnan-Taylor damping, β satisfies $0\leq\beta\leq 1/2$, and $m\geq 0$ is an integer. The function f is an external input. The concerned boundary conditions correspond to hinged boundary. The global dynamics which we are concerned are the existence of the global attractor,

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exponential attractor and flat inertial manifold for (1)–(3). The case was studied in [2] by Y. You when $n=1$, $m=0$, $\beta=0$, the case was studied in [3] by Y. You and M. Taboada when $n=1$ and the term $\delta\Delta^2 u_t$ is replaced by $\delta\Delta u_t$. In [2], [3], the authors did not give the existence of the global attractor and exponential attractor. In [4], A. Eden and A.J. Milani considered the global attractor, exponential attractor and flat inertial manifold for the following equation

$$\varepsilon u_{tt}+u_t+a\Delta^2 u=\Big(k\int_\Omega|\nabla u|^2-\beta\Big)\Delta u+f$$

where ε, a and β are positive constants. There, they use α-contraction map to obtain the global attractor. In this paper, we consider the problem of (1)–(3) in high dimensional case.

Let $H=L^2(\Omega)$, $V=H^2(\Omega)\cap H_0^1(\Omega)$, $Y=D(A)=\{u\in H^4(\Omega), u, \Delta u\in H_0^1(\Omega)\}$, and $E_0=V\times H$, $E_1=Y\times V$, we consider the solution $\{u(t),u_t(t)\}=S(t)\{u_0,u_1\}$ with values either in E_0, E_1. The global existence of such solutions is assured by

Theorem 1 *Assume* $\{u_0,u_1\}\in E_0$ *and* $f\in C(R^+;H)$. *Then there exists a unique solution* u *of* (1)–(3), *such that* $\{u,u_t\}\in C(R^+;E_0)$. *If in addition* $\{u_0,u_1\}\in E_1$ *and* $f\in C^1(R^+;H)$, *then* $\{u,u_t\}\in C(R^+;E_1)$.

Proof It is similar to the proof of [5], we omit it here.

This paper is organised as follows: In Section 2 and Section 3, we give the existence of a bounded absorbing set for $S(t)$ respectively in E_0 and E_1; in Section 4, we use the method of decomposition for operator $S(t)$ ([6]) to prove the existence of a compact global attractor in E_0 for $S(t)$; in Section 5 and Section 6, we give the existence of exponential attractor and inertial manifold for (1)–(3).

2. Absorbing Sets in E_0

We shall denote by $|\cdot|_0$ and $(\cdot,\cdot)$ the norm and inner product in H.

Multiplying (1) in H by u_t, we obtain

$$\frac{d}{dt}\Big(|u_t|_0^2+\alpha|\Delta u|_0^2+\frac{1}{2b}(a+b|\nabla u|_0^2)^2\Big)+2\delta|\Delta u_t|_0^2+2q\Big(\frac{1}{2}\frac{d}{dt}|\nabla u|_0^2\Big)^{2(m+\beta+1)}=2(f,u_t)\tag{4}$$

Since

$$|u_t|_0^2\le\lambda_1|\Delta u_t|_0^2$$

where λ_1 is the first eigenvalue of $(-\Delta)^2$ with (2), we have

$$\frac{d}{dt}\Big(|u_t|_0^2+\alpha|\Delta u|_0^2+\frac{1}{2b}(a+b|\nabla u|_0^2)^2\Big)+\delta|\Delta u_t|_0^2+2q\Big(\frac{1}{2}\frac{d}{dt}|\nabla u|_0^2\Big)^{2(m+\beta+1)}\le\frac{1}{\lambda_1\delta}|f|_0^2\tag{5}$$