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THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO THE DAMPED P-SYSTEM WITH BOUNDARY EFFECTS

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Abstract The initial boundary value problems (IBVP) for the P-system with damping on $[0,1]\times[0,+\infty)$ are considered. The global existence of smooth solutions for the IBVP are proved, and their large-time behavior is analyzed. The time-asymptotic equivalence of these solutions to the solutions of the IBVP for the reduced system (1.2) is shown.

Key Words P-system; initial boundary value problem; global existence; large-time behavior.

Classification 35L65, 35L45, 35L55.

1. Introduction

Consider the following P-system with damping in Lagrangian coordinates

$$\begin{cases} v_t - u_x = 0 \\ u_t + p_x = \alpha u, \quad \alpha > 0 \end{cases} \tag{1.1}$$

where $p = p(v)$ with $p'(v) < 0$. This system can be viewed as the Euler equations with friction term added to the momentum equation, and used to describe the compressible flow through porous media where $v > 0$ is the specific volume, u is the velocity, and $p(v)$ is the pressure.

By approximating $(1.1)_2$ with Darcy's law, we obtain the following systems

$$\begin{cases} v_{dt} = -\dfrac{1}{\alpha} p(v_d)_{xx} \\ u_d = -\dfrac{1}{\alpha} p(v_d)_x \end{cases} \tag{1.2}$$

It has been shown in [1] that the smooth solutions of the Cauchy problem for (1.1) with initial data

$$(v,u)(x,0) = (v_0,u_0)(x), \quad v_0(\pm\infty) = v_\pm$$

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tend time-asymptotically to the solutions of (1.2) with initial data

$$v_d(x,0) = v^*(x+b)$$

where v^* is the similarity solution to $(1.2)_1$, and b a constant. Namely, the nonlinear diffusive phenomenon of smooth solutions to (1.1) occurs due to the damping mechanism. Based on the resolution of perturbated Riemann problem for (1.1) in [2] and [3], the nonlinear diffusive phenomenon of entropy weak solutions to (1.1) were shown in [4]. There are other related results to [1], such as [5–9] for smooth solutions, and [4], [10–12] for weak solutions.

We are interested in the time-asymptotic equivalence of the hyperbolic system (1.1) to the reduced decoupled system (1.2) with boundary effects. In the present paper, we inverstigate the initial boundary value problems (IBVP) for (1.1) on $\Omega \equiv [0,1]\times[0,+\infty)$ with initial data

$$(v,u)(x,0) = (v_0,u_0)(x), \quad x \in [0,1] \tag{1.3}$$

and boundary values given by one of the followings

$$u(0,t) = u_1(t), u(1,t) = u_2(t), \qquad t \geq 0 \tag*{$(1.4)_1$}$$

$$u(0,t) = u_1(t), p(1,t) = p_2(t), \qquad t \geq 0 \tag*{$(1.4)_2$}$$

$$p(0,t) = p_1(t), p(1,t) = p_2(t), \qquad t \geq 0 \tag*{$(1.4)_3$}$$

where $u_i(t) \to 0$ $(i=1,2)$, as $t \to +\infty$.

Our aim is to verify if the smooth solutions to IBVP (1.1), (1.3) and (1.4) are time-asymptotically equivalent to those of (1.2) with initial data

$$v_d(x,0) = v_{d0}(x), \quad x \in [0,1] \tag{1.5}$$

and the corresponding boundary values given by

$$p_x(0,t) = f_1(t), p_x(1,t) = f_2(t), \qquad t \geq 0 \tag*{$(1.6)_1$}$$

$$p_x(0,t) = f_1(t),\ p(1,t) = p_2(t), \qquad t \geq 0 \tag*{$(1.6)_2$}$$

$$p(0,t) = p_1(t),\ p(1,t) = p_2(t), \qquad t \geq 0 \tag*{$(1.6)_3$}$$

respectively, where $f_i = -(u_i + du_i/dt)(t)$, $i = 1,2$.

First, we consider the IBVP (1.1), (1.3) and $(1.4)_1$ and the IBVP (1.2), (1.5) and $(1.6)_1$ respectively. For simplicity, we consider a typical case $p(v) = v^{-\gamma}$, $\gamma \geq 1$, and assume that $\alpha = 1$. In addition, we assume that the compatibility conditions of the initial and boundary values at (0, 0) and (1, 0) hold.

Integrating $(1.1)_1$ over $[0,1]\times[0,t]$, we obtain

$$\int_0^1 v(x,t)dx = \int_0^1 v_0(x)dx + \int_0^t (u_2-u_1)(\tau)d\tau \triangleq Q(t) \tag{1.7}$$