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A REMARK ON MONGE-AMPÈRE EQUATIONS IN NONSTRICTLY CONVEX DOMAINS

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Abstract In this paper, we discuss the regularity of the weak solutions and the existence of the classical solutions for Monge-Ampère equations on the bounded convex domains possessing uniform parabolic support. This paper improves the conclusion of [1].

Key Words Partial differential equation; elliptic different equation; Monge-Ampère equation; nonstrict convex domain.
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A vast study on Monge-Ampère equations in strictly or uniformly convex domains has been made, but there have been only a few results in nonstrictly convex domains. In [1], we discussed the Dirichlet problem for Monge-Ampère equations in nonstrictly convex domains with uniformly parabolic support

$$\det D^2u = f(x, u, Du), \quad u\,|_{\partial\Omega}= \varphi(x) \tag{1}$$

But the conditions (1.3–1.4) in [1] do not contain the results corresponding to the strictly convex domains (see [2]), therefore the conclusions of [1] ought to be made better. The conclusion of this paper just contains that one of [2], therefore it is better than the conclusions of [1].

The concept of parabolic support can be given as follows: Let Ω be a bounded C^1 domain in R^n, $y \in \partial\Omega$. We can choose the coordinate system for which y is the coordinate origin and the x_n-axis coincides with the inner normal of $\partial\Omega$ at y. Therefore $\partial\Omega$ can be represented by

$$x_n = X(x'), \quad x' = (x_1 \cdots, x_{n-1}) \tag{2}$$

in a neighbourhood of y. Thus we say that $\partial\Omega$ has a uniformly parabolic support of order $\tau \geq 0$ if there exist positive constants a and δ, such that

$$X(x') \geq 2a|x'|^{2+\tau}, \quad |x'| \leq \delta$$

In [1], the key results are concerned in the regularity of the weak solutions (Theorem 1) and the existence of the classical solutions (Theorems 4–5). Here we shall only improve the conclusion of Theorem 1 of [1]. Of course, the corresponding result with Theorems 4–5 of [1] should be held.

Theorem *Let Ω be a bounded C^1 domain in R^n having uniformly parabolic support of order $\tau \geq 0$, and let $\varphi \in C^{0,1}(\overline{\Omega}) \cap C^2(\Omega)$ a convex function, and $u \in C^0(\overline{\Omega})$ a convex weak solution of the Dirichlet problem (1). And suppose that the following conditions are satisfied:*

(1) $f \in C^1(\Omega \times R \times R^n)$ is a nonnegative function and

$$f_z(x,z,p) \geq 0, \quad \forall (x,z,p) \in \Omega \times R \times R^n \tag{3}$$

(2) There exists a neighbourhood N of $\partial\Omega$ and nonnegative constants α, β, such that

$$f(x,\varphi(x),p) \leq \mu \widetilde{d}(x)^\beta (1+|p|^2)^{\alpha/2}, \quad \forall (x,p) \in N \times R^n \tag{4}$$

where $\widetilde{d}(x) = \mathrm{dist}\,(x, \partial\Omega)$, and

$$\beta \geq \max\{\tau(n-1), \alpha - (n+1) + \tau(n-1)\} \tag{5}$$

Then $u \in C^{0,1}(\overline{\Omega})$ and

$$\sup |Du| \leq C \tag{6}$$

where C depends only on n, μ, α, β, τ, a, δ, $|u|_{0,\Omega}$ and $|\varphi|_{0,1,\Omega}$.

Proof As [1], we shall only study the inner normal derivative of u on the boundary. And because the conclusion for $\tau = 0$ has been proved in [2], here we shall only study the case $\tau > 0$.

We choose the coordinate system by the same way as above. Set

$$\psi(d) = -\frac{1}{\nu}\ln(1+Kd) \tag{7}$$

$$d = d(x) = ks^{-\tau/2} - \sqrt{d'} \tag{8}$$

where $k = 2^\tau/a$, $r = |x'|$, $s = r^2 + x_n^2$, $d' = r^2 + (x_n - ks^{-\tau/2})^2$, ν and K are the positive constants to be determined afterwards.

We consider the subset

$$D_y = \{x \in \Omega \mid d < d_0, r < \delta, s < \delta_1\}$$

where δ is the constant in the above definition of uniformly parabolic support, and δ_1 is small enough.

First, we demonstrate the following lemma.

Lemma *If d_0 is small enough, then $d(x) > 0$ in D_y and D_y is small enough, too.*