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DECAY PROPERTIES OF GLOBAL SOLUTIONS FOR BENJAMIN-ONO EQUATION OF HIGH ORDER

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Abstract The decay properties of global solutions for the Benjamin-Ono equation of high order are obtained as $|x| \to \infty$. An Iorio's type result is derived for this equation.
Key Words Global solution; weighted Sobolev spaces; Benjamin-Ono equation of high order; decay properties.
Classification 35Q.

1. Introduction

In this paper, we study the decay properties of global solutions for Benjamin-Ono (BO) equation of high order. In [1–3], the authors establish the existence and uniqueness of global solutions for the BO equation and its high order approximate form. The decay properties of the solution to the BO equation in Sobolev spaces $J_r = H^r(R) \cap L_r^2(R)$ are also obtained in [1]. Our aim in the present paper is to discuss the decay properties of the solution for the BO equation of high order and an Iorio's type result is derived. Before starting our work we introduce the definition of a weighted Sobolev space.

Definition 1.1 *Define $J_{r,s} = H^s(R) \cap L_r^2(R)$ with the following norm*

$$\|f\|_{J_{r,s}}^2 = \|f\|_s^2 + \|f\|_{L_r^2}^2, \quad f \in J_{r,s}$$

where $H^s(R)$ is the usual real Sobolev space and $L_r^2(R)$ is the collection of all measurable functions $f: R \to R$ such that

$$\|f\|_{L_r^2}^2 = \int (1+x^2)^r |f|^2 dx < +\infty$$

For simplicity, we write $J_r = J_{r,r}$. Obviously, $J_{r,s} \subset J_r$ and $J_{s,s} \subset H^s$ if $r \le s$.

Using the main result of Appendix A in [1] we can easily prove the following

Theorem 1.1 *If $f \in J_{r,s}, r \leq s$, then $x^\beta \partial_x^\alpha f \in L^2$ for all nonnegative integers α, β, $0 \leq \alpha + \beta \leq r$; and there exists a constant $C_{\alpha,\beta} > 0$ such that*

$$\left\| x^\beta \partial_x^\alpha f(x) \right\|_0 \leq C_{\alpha,\beta} \|f\|_{J_r} \leq C_{\alpha,\beta} \|f\|_{J_{r,s}}$$

Since the ideas and methods involved here are the same as those of [2], we shall only indicate the main points without going into much details in establishing our results. For further information on the BO equation of high order and symbols used here, we refer the reader to [2]. For convenience, we would like to list the following frequently used symbols in this paper as follows:

$$E_\mu(\xi, t) = \exp[-(\mu\xi^4 + 4i\xi^3)t]; \widehat{S_\mu(t)}\nu = E_\mu(\xi, t)\hat{\nu}(\xi)$$

The following two are the initial value problems for the BO equation of high order and its parabolic regularization.

$$\begin{aligned} &\partial_t u = -\partial_x(u^3 + 3uHu_x + 3H(uu_x) - 4u_{2x}) \\ &u(x, 0) = \varphi(x) \end{aligned} \tag{1.1}$$

$$\begin{aligned} &\partial_t u = -\mu\partial_x^4 u - \partial_x(u^3 + 3uHu_x + 3H(uu_x) - 4u_{2x}), \quad 0 < \mu < 1 \\ &u(x, 0) = \varphi(x) \end{aligned} \tag{1.2}$$

2. Main Results and Their Concise Proof

In this part we establish an Iorio's type result for problems (1.1), (1.2). The results show that the presence of higher order local dispersive term (and even higher order local dissipative terms) are not necessarily capable of taming the slow decay of solutions. In this case, the slow decay would be the effect of the non-local nonlinear terms.

Theorem 2.1 *Let $\mu \in (0, 1)$ be fixed, $\varphi \in J_{2,4}$. Then there exist a constant $T' > 0$ (depending only on $\|\varphi\|_{J_{2,4}}$), and a unique function $u_\mu \in C([0, T']; J_{2,4})$ such that*

$$u_\mu = S_\mu(t)\varphi - \int_0^t S_\mu(t - \tau)\partial_x \left(u_\mu^3 + 3u_\mu H(\partial_x u_\mu) + 3H(u_\mu \partial_x u_\mu) \right) d\tau$$

Proof First consider the following complete metric space

$$I_{2,4}(T) = \{f; f \in C([0, T]; J_{2,4}), \|f(t) - S_\mu(t)\varphi\|_{J_{2,4}} \leq \|\varphi\|_{J_{2,4}}, \ t \in [0, T]\}$$

The topology of $I_{2,4}(T)$ is induced by $\sup\limits_{[0,T]} \|f(t) - g(t)\|_{J_{2,4}}$ for $f(t), g(t) \in I_{2,4}(T)$

Define mapping A as follows:

$$Af(t) = S_\mu(t)\varphi - \int_0^t S_\mu(t - \tau)\partial_x \left(f^3 + 3fHf_x + 3H(ff_x) \right) d\tau$$