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THE INERTIAL FRACTAL SETS FOR NONLINEAR SCHRÖDINGER EQUATIONS*

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Abstract The existence of inertial fractal sets for weakly dissipative Schrödinger equations which possess (E_0, E) type compact attractor is proved. The estimates of the upper bounds of fractal dimension of inertial fractal set are also obtained.

Key Words Schrödinger inertial fractal set.
Classification 35Q55.

1. Introduction

In the study of the inertial manifold of the 2D Navier-Stokes equations (NSE) representing turbulent flows, one finds out that [2] since there exist spectral barriers and spectral gap conditions, the existence of an inertial manifold for 2D NSE is still a mystery. Recently, Eden et al.[3] have studied and discovered that some dissipative evolution equations with real coefficients, for which the (E, E) type compact attractors exist, including 2D NSE, have a kind of set similar to inertial manifold-inertial set. This paper advances the previous results to complex weakly infinite dimensional dynamical system that only possesses (E_0, E) type compact attractors.

2. Main Results

Let $D(A)$, V be two Hilbert spaces, $D(A)$ be dense in V and compactly imbedded into V.

We study

$$\frac{du}{dt} + Au + g(u) = f(x), \quad t > 0, \ x \in \Omega \tag{1}$$

$$u(0) = u_0 \tag{2}$$

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$$u|_{\partial\Omega} = 0 \tag{3}$$

where Ω is a bounded open set in R^n, $\partial\Omega$ is smooth. A is a positive self adjoint operator with a compact inverse. Let $\{w_n, n = 1, 2, \cdots\}$ denote the complete set of eigenvectors of A, the corresponding eigenvalues are

$$0 < \lambda_1 < \lambda_2 < \cdots < \lambda_n < \cdots \nearrow +\infty \tag{4}$$

We assume that the nonlinear semigroup $S(t)$ defined in (1)–(3) possesses a $(D(A), V)$ type compact attractor, namely, there exists a compact $\mathcal{A}$ in V, $\mathcal{A}$ attracts all bounded subsets in $D(A)$ and it is invariant under the action of $S(t)$.

Definition 1 *A compact set M in V is called an inertial fractal set of $(D(A), V)$ type for $(S(t), B)$ if $\mathcal{A} \subseteq M \subseteq B$ and*

1. *$S(t)M \subseteq M$, $\forall t \geq 0$,*
2. *M has finite fractal dimension, $d_F(M) < \infty$,*
3. *there exist positive constants c_0, c_1 such that*

$$\mathrm{dist}_V(S(t)B, M) \leq c_0 e^{-c_1 t}, \quad \forall t > 0$$

where $\mathrm{dist}_V(A, B) = \sup_{x\in A} \inf_{y\in B} |x - y|_V$, B is a positively invariant set for $S(t)$ in V.

Definition 2[3] *If for every $\delta \in \left(0, \frac{1}{8}\right)$, there exists an orthogonal projection P_{N_0} of rank equal to N_0 such that for every u and v in B, either*

$$|S(t_*)u - S(t_*)v|_V \leq \delta|u - v|_V \tag{5}$$

or

$$|Q_{N_0}(S(t_*)u - S(t_*)v)|_V \leq |P_{N_0}(S(t_*)u - S(t_*)v)|_V \tag{6}$$

Then we call $S(t)$ is squeezing in B, where $Q_{N_0} = I - P_{N_0}$.

Theorem 1 *Suppose (1)–(3) satisfies the following conditions*

1. *there exists a $(D(A), V)$ type compact attractor $\mathcal{A}$.*
2. *there exists a compact set B in V which is positively invariant for $S(t)$.*
3. *$S(t)$ is squeezing and Lipschitz continuous, that is there exists a bounded function $l(t)$ such that $|S(t)u - S(t)v|_V \leq l(t)|u - u|_V$ for every u, v in B.*

Then (1)–(3) admits a $(D(A), V)$ type inertial fractal set M for $(S(t), B)$ and

$$M = \bigcup_{0\leq t\leq t_*} S(t)M_* \tag{7}$$

where

$$M_* = \mathcal{A} \cup \left(\bigcup_{j=1}^{\infty} \bigcup_{k=1}^{\infty} S(t_*)^j \left(E^{(k)}\right) \right) \tag{8}$$