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WELL-POSED PROPERTY OF THE INVERSE PROBLEM FOR THE LAME'S PARAMETERS λ, μ AND DISSIPATIVE FACTOR γ *

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Abstract In this paper, the inversion of coefficients Lame's parameters λ, μ and dissipative factor γ in one order dissipative elastic wave equation in the vector form is discussed. Under some smoothness conditions, we have proved the existence, uniqueness and extension theorems of the local solution to the inverse problem by means of an equivalent integral system. Stability, well-posed property of the solution and the property of the solution at the end point of the largest existence interval are also researched.
Key Words Inverse problem; elastic wave; Lame's parameter; dissipative factor; largest existence interval; well-posed property.
Classification 35R30.

1. Transformation and Decomposition of the Basic Model

It is well-known that there are many inverse methods for the parameters of medium characteristics in plane stratified medium [1–4], but few works concerning dissipative effection because of the complexity. In this paper, starting with 2–**D** one order dissipative elastic wave equation in the vector form, we are going to set up the inverse method for the Lame's parameters λ, μ and dissipative factor γ in plane stratified medium simultaneously. Through transforming and decomposing the inverse problem for λ, μ, γ in 2–**D** one order dissipative elastic wave equation in the vector form into the inverse problem for λ, μ, γ in 1–**D** one order partial differential system with a free boundary, we shall prove the inverse problem well-posed in theory.

Supposing the earth is a linear isotropic elastic semi-infinite plane stratified medium and has only small displacement no hypocentre inside it, we may study the elastic wave problem in a plane which is perpendicular to the surface of the earth for the convenience. In the state of strain, 2–**D** dissipative elastic wave equation which describes the process of elastic oscillation propagation in the plane can be written in the following one order vector form [5]

$$\left(A\frac{\partial}{\partial t} - B_1\frac{\partial}{\partial x_1} - B_2\frac{\partial}{\partial x_2} + B_3\right)\vec{U} = 0, \quad x_1 > 0,\ t > 0 \tag{1}$$

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where $\vec{U} = (u_1, u_2, \sigma_{11}, \sigma_{12}, \sigma_{22})'$, (t, x_1, x_2) is time-point coordinate system, $x_1 > 0$ denotes the depth into the ground with $x_1 = 0$ the ground surface, $(u_1, u_2)'$ particle velocity vector, σ_{ij}, $i, j = 1, 2$ stress tensor, $a = (2\mu)^{-1}$, $b = \lambda[4\mu(\lambda+\mu)]^{-1}$,

$$A = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 & 0 \\ 0 & 0 & a-b & 0 & -b \\ 0 & 0 & 0 & 2a & 0 \\ 0 & 0 & -b & 0 & a-b \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad B_3 = \begin{bmatrix} \gamma & 0 & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Lame's parameters λ, μ and dissipative factor γ are the functions of only x_1 in the plane stratified medium $\{(x_1, x_2) \mid x_1 > 0\}$, the density ρ is supposed to be constant 1 for simplicity.

The initial condition is taken of form

$$\vec{U}\mid_{t<0} = 0, \quad x_2 \in (-\infty, \infty) \tag{2}$$

The boundary conditions

$$\sigma_{1j}\mid_{x_1=0} = \delta(x_2, t), \quad t \geq 0,\ j = 1, 2 \tag{3}$$

are called the coupling pulse waves source conditions, $\delta(x_2, t)$ is Dirac function and the boundary conditions

$$u_j\mid_{x_1=0} = g_j(x_2, t), \quad t \geq 0, -\infty < x_2 < \infty,\ j = 1, 2 \tag{4}$$

are called the additional conditions which are particle velocities measured on the line $\{(x_1, x_2) \mid x_1 = 0\}$ in two directions of the coordinate axes ox_1 and ox_2.

These equations (1)–(4) form a nonlinear inverse problem for three unknown functions $\lambda(x_1), \mu(x_1)$ and $\gamma(x_1)$ from the particle velocity fields $u_1(0, x_2, t), u_2(0, x_2, t)$ due to the coupling transverse and longitudinal pulse waves simultaneously at normal incidence $\sigma_{1j}\mid_{x_1=0} = \delta(x_2, t)$, $t \geq 0$, $j = 1, 2$. It is the basic model in the paper.

Let $\widetilde{U}(x, v, t)$ be the Fourier transformation of $\vec{U}(x_1, x_2, t)$ with respect to x_2, $k_s = \sqrt{\mu}$, $k_p = \sqrt{\lambda + 2\mu}$, $\eta = \int_0^{x_1} k_s(x_1)dx_1$, $\widetilde{U} = R\vec{V}$, $\beta_1(\eta) = \gamma(\eta) + k_s^{-1}\frac{dk_s}{d\eta}$, $\beta_2(\eta) = \gamma(\eta) + k_s^{-1}\frac{dk_p}{d\eta}$, $\beta_3(\eta) = k_s^{-1}\frac{d}{d\eta}(k_p - 2k_s)$, $i = \sqrt{-1}$