J.Partial Diff. Eqs., 7(1994), 171–182
©International Academic Publishers Vol.7, No.2

EXISTENCE OF GLOBAL AND PERIODIC SOLUTIONS FOR DELAY EQUATION WITH QUASILINEAR PERTURBATION

Xiang Xiaoling
(Department of Mathematics, Guizhou University, Guiyang 550025)
(Received July 6. 1991; revised June 14, 1993)

Abstract Delay parabolic problems have been studied by many authors. Some authors investigated more general delay problem (refer to [1], [2]), some investigated concrete delay partial differential equations. Recently, we have done some work on delay parabolic problem. We discussed semilinear parabolic delay problem and obtained some results on the existence of solutions. In particular the results on existence of periodic solutions are characteristic (see [3], [4], [5], [6]).

The purpose of this paper is to study delay equation with quasilinear perturbation. We present the existence of global and periodic solutions of abstract evolution equations in Section 2. The abstract results are used to obtain the existence of global and periodic solutions of delay parabolic problem with quasilinear perturbation in Section 3. We make preparation for our investigation and give a generalization of Gronwall inequality (Lemma 1.3) which is used in next section.

1. Preparation

If X and Y are Banach spaces such that X is continuously imbedded in Y then we write $X \hookrightarrow Y$; X is compactly imbedded in Y, then we write $X \hookrightarrow\hookrightarrow Y$. Let A be a linear operator in some Banach spaces. We denote by $R(\lambda, A)$ the resolvent of complexification of A. $L(X, Y)$ denotes the Banach space which consists of all bounded linear operators from X to Y and equipped with operator norm.

Let X be a Banach space and let T be a fixed positive number. Suppose that:

(A1) $\{A(t)|0 \le t \le T\}$ is a family of closed densely defined linear operators in X such that the domain $D(A(t))$ of $A(t)$ is independent of t.

(A2) For each $t \in [0, T]$ the resolvent $R(\lambda, A(t))$ exists for all λ with $\mathrm{Re}\lambda \le 0$ and

$$\|R(\lambda, A(t))\| \le C(1+|\lambda|)^{-1}$$

where C is some constant that is independent of λ and t.

These assumptions imply that $A(t)$ has an inverse $A^{-1}(t) \in L(X.X)$. Write $A(0) = A, \|x\|_1 = \|Ax\|(x \in D(A))$, consequently $X_1 = (D(A), \|\cdot\|_1)$ is a Banach space and $X_1 \hookrightarrow X$.

(A3) The map $A(\cdot):[0,T]\to L(X_1,X)$ is Hölder continuous.

Assumption (A1) and (A2) imply that $-A(t)$ is the infinitesimal generator of a holomorphic semigroup $\{e^{-\tau A(t)}|0\le\tau<+\infty\}$ in $L(X,X)$. We introduce fractional operator $A^\alpha(t)(\alpha\in(0,1])$ in usual way (refer to [7], [8], [9]). Which has dense domain and we have that $D(A^\alpha(t))\hookrightarrow D(A^\beta(t))$ for $\alpha\ge\beta>0$. Let $\|x\|_\alpha=\|A^\alpha x\|(x\in D(A^\alpha))$. $X_\alpha=(D(A^\alpha),\|\cdot\|_\alpha)$ is a Banach space and $X_\beta\hookrightarrow X_\alpha(0\le\alpha\le\beta\le1)$.

Consider linear initial value problem

$$\dot u+A(t)u=q(t)\qquad 0\le t\le T \tag{1.1a}$$

$$u(0)=x \tag{1.1b}$$

with $g\in C([0,T],X)$ and $x\in X$. By a solution u of (1.1) we mean a function $u\in C([0,T],X)\cap C^1((0,T],X)$ with $u(0)=x, u(t)\in D(A)$ for $t>0$ and $\dot u(t)+A(t)u(t)=g(t)$ for $0<t\le T$. Our assumptions imply the following theorem:

Theorem 1.1 (1.1) *has a unique solution u for every Hölder continuous right-hand side g. Moreover $u\in C^1([0,T],X)$, provided $x\in D(A)$.*

There exists a unique evolution operator $U(t,\tau)\in L(X,X), 0\le\tau\le t\le T$, such that every solution u of (1.1) can be represented in the form:

$$u(t)=U(t,0)x+\int_0^t U(t,\tau)g(\tau)d\tau\qquad 0\le t\le T$$

The operator $U(t,\tau)$ is strongly continuous on the closure of the set $\Delta=\{(t,\tau)\in[0,T]^2|0\le\tau<t\le T\}$ and satisfies $U(t,t)=id_x, U(s,t)\cdot U(t,\tau)=U(s,\tau)(0\le\tau\le t<s\le T)$.

In the following lemma we collect the most important regular properties of the evolution operator. For abbreviation we denote the norm in $L(X_\alpha,X_\beta)$ by $\|\cdot\|_{\alpha,\beta}$.

Lemma 1.2 (1) *Suppose $0\le\alpha\le\beta\le1$, then*

$$\|U(t,\tau)\|_{\alpha,\beta}\le C(\alpha,\beta,\gamma)(t-\tau)^{-\gamma}$$

for $\beta-\alpha<\gamma<1$ and $0\le\tau<t\le T$. If $0\le\beta<\alpha<1$, then

$$\|U(t,\tau)\|_{\alpha,\beta}\le C(\alpha,\beta)$$

(2) *Suppose that $0\le\alpha<\beta\le1$, then*

$$\|U(t,\tau)-U(s,\tau)\|_{\beta,\alpha}\le C(\alpha,\beta,\gamma)|t-s|^\gamma$$

for $0\le\gamma<\beta-\alpha$ and $(t,\tau),(s,\tau)\in\bar\Delta$.

(3) *Let $0\le\alpha<1, 0\le\sigma<T$, and $g\in C([\sigma,T],X)$ then*

$$\Big\|\int_\sigma^t U(t,\tau)g(\tau)d\tau-\int_\sigma^s U(s,\tau)g(\tau)d\tau\Big\|_\alpha$$

$$\le C(\alpha,\beta)|s-t|^\gamma\max_{\sigma\le\tau\le T}\|g(\tau)\|$$