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THE STABILITY AND HOPF BIFURCATION OF THE PREY-PREDATOR SYSTEM WITH DELAY AND MIGRATION*

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Abstract In this paper we first investigate the system with the influence of delay and migration and give a theoretical analysis of the alternative change of the stability discovered by Stepán with computer program, then we reduce the system with the center manifold theorem and present an approximation form of Hopf bifurcation solutions. Finally we give the numerical analysis of stability for a concrete periodic solution.
Key Words Delay; migration; alternative change of stability; center manifold; Hopf bifurcation.
Classification 35K57.

1. Introduction

The Prey-Predator systems have been interesting many ecologists and mathematicians ([1–5]). Moreover the periodic phenomena of the systems are the focus of study. In this paper we first investigate the systems with the influence of delay and migration, and give a theoretical analysis of the alternative change of the stability discovered by Stepán with computer program; then we reduce the systems with the center manifold theorem, and present an approximation form of Hopf bifurcation solutions. Finally we give the numerical analysis of stability for a concrete periodic solution. The Prey-Predator model assumes the form

$$
\begin{aligned}
&\frac{\partial N}{\partial t} = d\frac{\partial^2 N}{\partial x^2} + \varepsilon N\Big(1 - \frac{N}{K} - \frac{\alpha}{\varepsilon}P\Big), \qquad 0 < x < \pi,\ t \in R\\
&\frac{\partial P}{\partial t} = d\frac{\partial^2 P}{\partial x^2} - \gamma P + \beta P\int_0^\infty N(x, t-\tau)w(\tau)d\tau\\
&N(0,t) = N(\pi,t) = \frac{\gamma}{\beta}\\
&P(0,t) = P(\pi,t) = \frac{\varepsilon}{\alpha}\Big(1 - \frac{\gamma}{K}\Big)
\end{aligned}
\tag{1}
$$

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where N and P are the population densities of the prey and the predator respectively; the parameters d, K, α, β, γ and ε are positive constants, d the migration rate, K the carrying capacity of the prey, α the rate of predation per predator, β the rate of conversion of prey into predator, γ the specific mortality of predator in absence of prey, ε the specific growth rate of prey at zero density in absence of predators. With the help of the scalar weight function $w(\tau)$, the model takes the density of the prey in the past with $\int_0^\infty w(\tau)d\tau = 1$ into account. Since this paper discusses the stability of equilibrium and Hopf bifurcation, the influence of the initial value is not important, and it will not mentioned afterwards. Let us take the transformation

$$u = N - \frac{\gamma}{\beta}, \quad v = P - \frac{\varepsilon}{\alpha}\Big(1 - \frac{\gamma}{K\beta}\Big)$$

For determination, let the weight function be $w(\tau) = 1/2[\delta(0) + \delta(r)]$, where r indicate the delay amount. Similar to that in [1], let the parameters be $\alpha = \beta = \gamma = \varepsilon = 1$, $K = 2$,then the equations assume the form

$$\begin{aligned} u_t &= du_{xx} - \frac{1}{2}u - v - \frac{1}{2}u^2 - uv \\ v_t &= dv_{xx} + \frac{1}{4}u + \frac{1}{4}u(x, t-r) + v\Big(\frac{1}{2}u + u(x, t-r)\Big) \\ u(0, t) &= u(\pi, t) = v(0, t) = v(\pi, t) = 0 \end{aligned} \tag{2}$$

In the discussion below, r is considered as a parameter. For the sake of covenience, after the usual time transformatin $t = r\bar{t}$, (2) has the form

$$\begin{aligned} u_t &= rdu_{xx} - \frac{r}{2}u - rv - \frac{r}{2}u^2 - ruv \\ v_t &= rdv_{xx} + \frac{r}{4}[u + u(x, t-1)] + \frac{r}{2}v[u + u(x, t-1)] \\ u(0, t) &= u(\pi, t) = v(0, t) = v(\pi, t) = 0 \end{aligned} \tag{3}$$

In these equations, we always agree on that the function valuation will be taken at (x, t) when we do not indicate the variable. We will analyse the law of the stability change of the trivial solution of (3), and discuss the existence, approximation expression and the stability of Hopf bifurcation solutions. Similar to the method used in [1, 5, 6, 7, 8], we transform the problem (3) into the problem of the abstract operator differential equation first.

2. Abstract Operator Differential Equation

Let X denote $\left\{\begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} \Big| f_i \in L^p(0, \pi), i = 1, 2\right\}$, the $\|\cdot\|_X = \sum_{i=1}^{2} \|f_i\|_p$ or some equivalence norm are equipped with X. let $\mathcal{B}$ denote

$$\left\{\begin{pmatrix} f_1(x,\theta) \\ f_2(x,\theta) \end{pmatrix} \Big| \forall\theta \in [-1, 0], \begin{pmatrix} f_1(x,\theta) \\ f_2(x,\theta) \end{pmatrix} \in X \text{ and } \|\cdot\|_X \text{ is continuity for } \theta\right\}$$