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STEFAN PROBLEM WITH CHANGE DENSITY UPON CHANGE OF PHASE (I)

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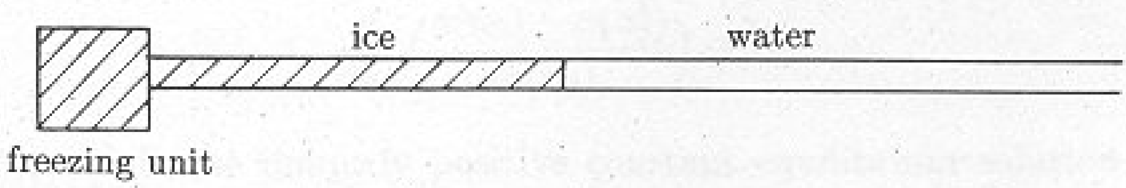
Abstract In this paper, we establish the existence of one-dimensional classical solution of one-phase problem and its continuous dependence. In addition, we prove that if $\varepsilon \to 0$, the free boundary $X(t)$ withdraws and solution converges to the solution of classical Stefan problem. The two-phase problem will be discussed in the coming paper.

Key Words asymptotic behavior; existence; phase transition problem.
Classification 35R35.

In the classical two-phase Stefan problem, the densities of two phases are always assumed to be equal. This allows one to neglect certain mechanical and thermomechanical questions, since the change in density during phase transition inevitably gives rise to the movement of material and development of thermal stresses in solid phase. However, neglecting heat convective, material movement in multiphase transition, and considering conductivity only, lead in general to serious incompatibility between mathematical model and physical nature.

V. Alexiades and J.B. Drake (1990[1]) considered a phase transition between ice and water in a infinite length of fine tube with a freezing unit on the left (see the figure below). Supposing ice and water are incompressible, they obtained the following mathematical model.



(Fig.1)

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$$\begin{cases} \rho_s c_p^s T_t = k_s T_{xx}, & 0 < x < X(t), 0 < t < T \\ T(0,t) = g(t), & 0 < t < T \\ T = T_m \quad \text{on } x = X(t) \\ T(x,0) = \psi(x), & 0 < x < a \\ X(0) = a, \quad \rho_s L X' - \frac{1}{2}\rho_s\Big(1 - \frac{\rho_s}{\rho_L}\Big)^2 X'^3 = k_s T_x(X(t),t) \end{cases}$$

Where ρ_s is ice density, L latent heat required to melt ice, $X(t)$ free boundary, k_s solid head conductivity, c_p^s specific heat under constant pressure T_m critical temperature. There is not any analytical result on this problem because of the cubic term in Stefan condition.

For the sake of convenience, we consider the following problem,

$$(P)\begin{cases} T_t = T_{xx}, \quad 0 < x < X(t), 0 < t < T & (1.1) \\ T(0,t) = g(t), \quad 0 < t < T & (1.2) \\ T = 0, \quad x = X(t) & (1.3) \\ T(x,0) = \psi(x) & (1.4) \\ X(0) = a, \quad X' - \varepsilon X'^3 = T_x(X(t),t) & (1.5) \end{cases}$$

In this paper, we acquire its existence of global classical solution, continuous dependence, monotonicity of free boundary $X(t)$ with respect to ε. We also prove that the solution converges to the solution of classical Stefan problem as $\varepsilon \to 0$.

A: Existence

Lemma 2.1 *For the fixed-boundary problem*

$$(P')\begin{cases} T_t - T_{xx} = 0, & 0 < x < X(t), 0 < t < T \\ T(0,t) = g(t) \\ T = 0 & \text{on } x = X(t) \\ T(x,0) = \psi(x), & 0 < x < a \end{cases}$$

if $\psi \in C^2[0,a], g(t), X(t) \in C^1[0,T], a > 0, \psi(a) = 0, \psi(0) = g(0)$, *then there exists uniquely solution* $T(x,t) \in C^{1+1,0+1}(\bar{Q}) \cap C^\infty(Q)$, $Q = \{(x,t), 0 < x < X(t), 0 < t < T\}$, *and satisfies the estimates*

(1) $|T(x,t)| \le M_0, \quad M_0 = \max(\|g\|_{C^0}, \|\psi\|_{C^0})$ (2.1)

(2) *If* $X(t)$ *is monotonically increasing,*

$$|T_x(X(t),t)| \le \max\Big(\frac{|g(t)|}{a}, \|\psi'\|_{C^0}\Big) \quad (2.2)$$