J. Partial Differential Equations
Vol. 1, *No.* 2, *Series A*(1988), 71—81

STATIC SOLUTIONS OF MIXED BURGERS-KdV EQUATION I

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(Received January 19, 1987; revised August 28, 1987)

Abstract

We show that, for given positive parameters β and γ, the Burgers-KdV equation (1.5) has infinitely many static solutions satisfying given boundary conditions (2.1) or (3.3), and that these solutions differ from each other mainly by their numbers of extremum points.

1. Introduction

It is well-known that the simplest model of the dissipation phenomenon can be discribed with the Burgers equation

$$u_t + uu_x - \gamma u_{xx} = 0, \quad (\gamma > 0) \tag{1.1}$$

which has the famous shock wave solution

$$u = \lambda + u'_\infty \tanh[(-u'_\infty/2\gamma)x'] \tag{1.2}$$

where $x' = x - \lambda t$, λ is the wave velocity, $u'_\infty = u_{+\infty} - \lambda$ (see fig. 1.1).

The simplest model on the dispersion phenomenon can be discribed with the KdV equation

$$u_t + uu_x + \beta u_{xxx} = 0 \tag{1.3}$$

which has the famous soliton solution (Fig. 1.2)

$$u = u_\infty + \varepsilon \operatorname{sech}^2\{(12\beta)^{-1/2}[\varepsilon^{1/2}(x - u_\infty t) - \varepsilon^{3/2}/3t]\} \tag{1.4}$$

where $x' = x - (u_\infty + \varepsilon/3)t$ (see Fig. 1.2).

Naturally, the simplest model on the mixed effect of the dissipation and the dispersion of a nonlinear wave can be discribed with the mixed Burgers-KdV equation (see[1], [2])

$$u_t + uu_x - \gamma u_{xx} + \beta u_{xxx} = 0 \tag{1.5}$$

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† The work of this author is supported by the Science and Engineering Research Council of the United Kingdom under the Grant GR/D/66340 awarded to the University of Birmingham, England where this research work was conducted.

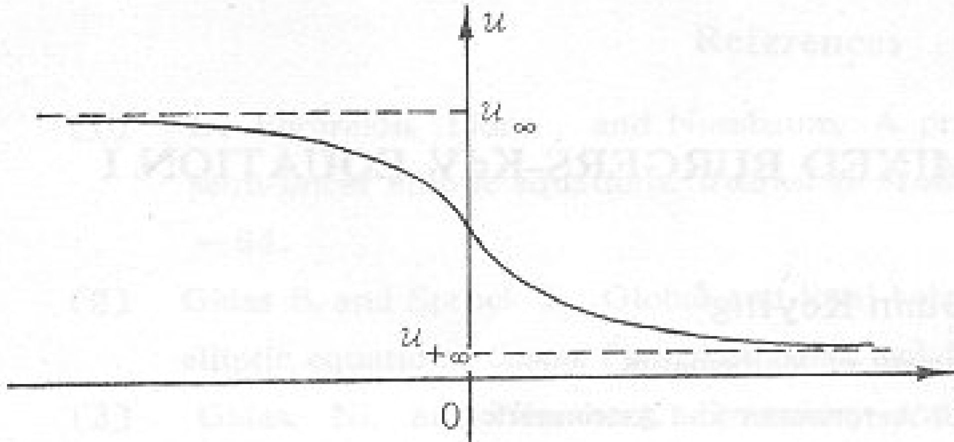

Figure 1.1

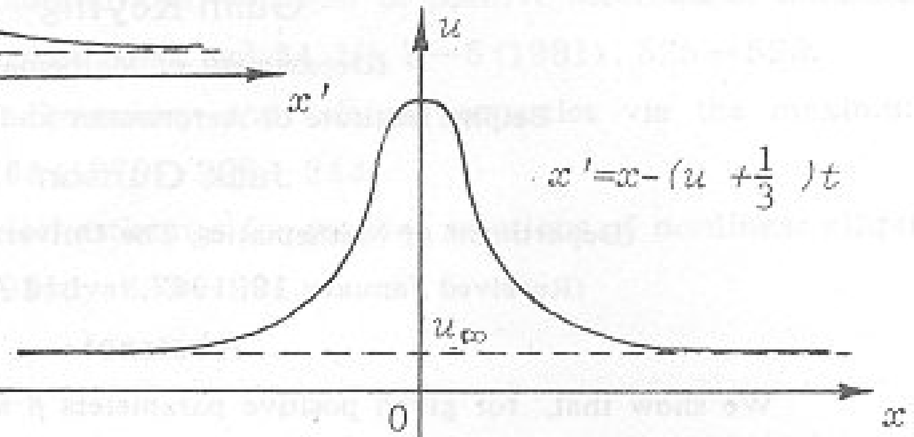

Figure 1.2

In recent years, in order to explain the phenomena of order and chaos in the fluid dynamics, the terms of the third derivatives are suggested to be added to the classical Navier-Stokes equations by some authors (see [3]). If the space is one dimensional, and if the external force and the pressure in the fluid can be neglected, then the new equation system of motion will be reduced to the mixed Burgers-KdV equation (1. 5).

This equation has already attracted much attention both in the field of theoretical research and in the field of application (see [1] — [6]). It has been proved that (1. 5) has a traveling wave solution, which tends to the shock wave solution of (1. 3) when $\beta \to 0$, and which tends to the soliton solution of (1. 4) when $\gamma \to 0$ (see [1], [6]).

However, since (1. 5) does not belong to any class of integrable nonlinear partial differential equations, in comparison with other famous nonlinear wave equations, only few properties of this equation has been studied very well, though the importance of this equaton is evident and has been noticed for many years.

Besides the difficulty on the integrability, the boundary conditions of the solutions are also a difficult problem to this equation. In fact, besides the global traveling wave, any practical process including both of the dissipative effect and the dispersive effect can only happen in a finite region of space. Since this equation has a third derivative with respect to the space variable x, in order to define a well posed initial value and boundary value problem, generally speaking, three boundary values are needed. But, up to now, it has still been being difficult to give three reasonable boundary conditions at the two ends of a finite interval. It has been shown that the properties of the solutions depend on the assignment of three boundary conditions sensitively (see [4]).