Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.2011.m1025

Vol. **4**, No. 3, pp. 359-378
August 2011

Dynamic Evasion-Interrogation Games with Uncertainty in the Context of Electromagetics

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Received 5 August 2010; Accepted (in revised version) 22 November 2010

Available online 20 June 2011

Abstract. We consider two player electromagnetic evasion-pursuit games where each player must incorporate significant uncertainty into their design strategies to disguise their intension and confuse their opponent. In this paper, the evader is allowed to make dynamic changes to his strategies in response to the dynamic input with uncertainty from the interrogator. The problem is formulated in two different ways; one is based on the evolution of the probability density function of the intensity of reflected signal and leads to a controlled forward Kolmogorov or Fokker-Planck equation. The other formulation is based on the evolution of expected value of the intensity of reflected signal and leads to controlled backward Kolmogorov equations. In addition, a number of numerical results are presented to illustrate the usefulness of the proposed approach in exploring problems of control in a general dynamic game setting.

AMS subject classifications: 35Q61, 83C50, 83C22, 65M32

Key words: Electromagnetic evasion-pursuit, uncertainty, feedback control, computational methods, Kolmogorov and Fokker-Planck equations.

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1. Introduction

In an electromagnetic evasion-interrogation game, the evader wishes to minimize the intensity of the reflected signal to remain undetected in carrying out his mission while the interrogator wishes to maximize the intensity of reflected signal to detect the attacker. It was demonstrated in [9] that it is possible to design ferroelectric materials with appropriate dielectric permittivity and magnetic permeability to significantly attenuate reflections of electromagnetic interrogation signals from highly conductive targets such as airfoils and missiles. These results were further sharpened and illustrated in [10] where a series of different material designs were considered to minimize over a given set of input design frequencies the maximum reflected field from input signals. In addition, it was shown

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that if the evader employed a simple counter interrogation design based on a fixed set (assumed known) of interrogating frequencies, then by a rather simple counter-counter interrogation strategy (use of an interrogating frequency little more than 10% different from the assumed design frequencies), the interrogator can easily defeat the evader's material coatings counter interrogation strategy to obtain strong reflected signals. From the combined results of [9, 10] it is thus rather easily concluded that the evader and the interrogator must each try to confuse the other by introducing significant *uncertainty* in their design and interrogating strategies, respectively.

Static two-player non-cooperative games with uncertainty were considered in [6]. In these problems, the evader and the interrogator are each subject to uncertainties as to the actions of the other. The evader wants to choose a best coating design (i.e., best dielectric permittivities and magnetic permeabilities) while the interrogator wants to choose best angles of interrogation and interrogating frequencies for input signals. Each player must act in the presence of incomplete information about the other's action. Partial information regarding capabilities and tendencies of the adversary can be embodied in probability distributions for the choices to be made. That is, one may formalize this by assuming the evader may choose (with an as yet to be determined set of probabilities) dielectric permittivity and magnetic permeability parameters from given admissible sets while the interrogator chooses angles of interrogation and interrogating frequencies from appropriate admissible sets . The formulation in [6] is based on the *mixed strategies* proposals of von Neumann [2, 30, 31] and the ideas can be summarized as follows. The evader does not choose a single coating, but rather has a set of possibilities available for choice. He only chooses the probabilities with which he will employ the materials on a target. This, in effect, disguises his intentions from his adversary. By choosing his coatings randomly (according to a best strategy to be determined in, for example, a minmax game), he prevents adversaries from discovering which coating he will use–indeed, even he does not know which coating will be chosen for a given target. The interrogator, in a similar approach, determines best probabilities for choices of frequency and angle in the interrogating signals. Note that such a formulation tacitly assumes that the adversarial relationship persists with multiple attempts at evasion and detection.

The problems are mathematically formulated in [6] as two sided optimization problems over spaces of probability measures, i.e., minmax games over sets of probability measures. That work demonstrates the feasibility and the potential usefulness of developing theories for problems with uncertainty. In this paper, we move toward a more realistic dynamic modeling by introducing time dynamics into the problem for single evasion attempts. Specifically, we allow a single evader to make dynamic changes to his dielectric permittivity strategies in response to feedback entailing measures of the reflection signals based on dynamic information with uncertainty about the interrogator's choices. Thus, this new formulation is more in the spirit of the deterministic dynamical differential games as formulated, for example, in [20] except here uncertainties of the two players' actions are a major feature as in the static games of [6, 31]. The remainder of this paper is organized as follows. We begin in Section 2 by presenting a description of our problem formulation. We then outline a theoretical and computational framework in Section 3 that provides a