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Implicit Shape Reconstruction of Unorganized Points Using PDE-Based Deformable 3D Manifolds

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Abstract. In this work we consider the problem of shape reconstruction from an unorganized data set which has many important applications in medical imaging, scientific computing, reverse engineering and geometric modelling. The reconstructed surface is obtained by continuously deforming an initial surface following the Partial Differential Equation (PDE)-based diffusion model derived by a minimal volume-like variational formulation. The evolution is driven both by the distance from the data set and by the curvature analytically computed by it. The distance function is computed by implicit local interpolants defined in terms of radial basis functions. Space discretization of the PDE model is obtained by finite co-volume schemes and semi-implicit approach is used in time/scale. The use of a level set method for the numerical computation of the surface reconstruction allows us to handle complex geometry and even changing topology, without the need of user-interaction. Numerical examples demonstrate the ability of the proposed method to produce high quality reconstructions. Moreover, we show the effectiveness of the new approach to solve hole filling problems and Boolean operations between different data sets.

AMS subject classifications: 65D18, 65F10, 65N08

Key words: Shape reconstruction, RBF interpolation, PDE diffusion model, segmentation.

1. Introduction

The shape reconstruction problem has several applications in areas that include computer visualization, data analysis, biomedical imaging, virtual simulation, computer graphics, etc. Surface reconstruction consists of finding out the original surface shape from partial information that can include points, pieces of curves, or surfaces. In this paper we consider implicit surface reconstruction from unorganized, eventually incomplete data

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sets. The reconstruction of a 3D model is usually obtained from scattered data points sampled from the surface of the physical 3D object. The acquisition of the data is in general realized through 3D scanning systems which are able to capture a dense sampling usually organized into range images, i.e. sets of distances to the data. The problem is ill-posed, i.e. there is no unique solution. Furthermore the problem can become quite challenging if the topology of the real surface is complicated or even unknown. A good reconstruction algorithm should be able to deal with non-uniform and incomplete data set in order to construct an arbitrary topology surface with a controlled hole filling strategy, smoothness and a right behavior for deformation, animation and other dynamical operation.

In general, the techniques for reconstructing a surface from unorganized data sets, can be divided into two main classes according to the way used to represent the surface: implicit (non-parametric approach) or explicit (parametric approach). Explicit surfaces in $\mathbb{R}^3$ are defined as the set of points $\{x(u,v), y(u,v), z(u,v)\}$ where the parameters $u, v \in \mathbb{R}$.

Very popular parametric approaches are based on spline patches which give rise to smooth reconstructed surface. However these approaches require a nice parametrization and arbitrary topologies have to be handled by patching of different surface pieces, which can be very difficult for surface reconstruction from arbitrary data in three or higher dimensions [3, 14, 26].

Several algorithms of shape reconstruction follow a computational geometry approach and are mainly based on Delaunay triangulations and Voronoi diagrams. These methods construct a collection of simplexes that form a shape from a set of unorganized points. These approaches are versatile, but it's not trivial to handle non-uniform and noisy data [1].

Recently, implicit methods have captured a lot of attention. In the implicit representation, the geometry and topology of the surface is defined as a particular iso-contour of a scalar implicit function over $\mathbb{R}^3$. The implicit approaches reconstruct an implicit function such that a certain level set of this function fits the data best. The iso-contour represents the reconstructed surface. The advantages of these techniques are the topological flexibility, the possibility to easily capture geometry property of the surface, and to realize in a simple way the classical Boolean operations, while it is a challenge to deal with open or incomplete surfaces, [21].

Approaches based on moving least-squares technique have been used widely for the reconstruction of point-set surfaces, see [2, 16, 34].

A particularly powerful approach for implicit surface reconstruction based on Radial Basis Functions (RBF) has been relatively recently introduced to deal with large scattered data sets without relying on a priori knowledge of the object topology [4, 5, 8, 30]. The implicit signed distance function is constructed by approximation or interpolation of surface and exterior constraint points defined by the data set. The coefficients of the reconstructed function are determined by solving linear systems of equations. The computational cost is very high for large data sets, when the construction is global. These approaches are not particularly robust in the presence of holes and low sampling density, ad hoc techniques and a priori knowledge on the surface topology should be inferred to obtain a faithful reconstruction [7].