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A Variational Binary Level Set Method for Structural Topology Optimization

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Abstract. This paper proposes a variational binary level set method for shape and topology optimization of structural. First, a topology optimization problem is presented based on the level set method and an algorithm based on binary level set method is proposed to solve such problem. Considering the difficulties of coordination between the various parameters and efficient implementation of the proposed method, we present a fast algorithm by reducing several parameters to only one parameter, which would substantially reduce the complexity of computation and make it easily and quickly to get the optimal solution. The algorithm we constructed does not need to re-initialize and can produce many new holes automatically. Furthermore, the fast algorithm allows us to avoid the update of Lagrange multiplier and easily deal with constraints, such as piecewise constant, volume and length of the interfaces. Finally, we show several optimum design examples to confirm the validity and efficiency of our method.

AMS subject classifications: 65K10, 74B05, 74S05

Key words: Structural optimization, binary level set method, finite element method.

1 Introduction

The topology optimization of continuum structures, as one of the most challenging tasks, has been widely investigated in the relevant literatures [1, 2, 24, 26, 29, 32, 35]. The homogenization method, which was first developed by Bendsoe and Kikuchi (see [3]), is a popular and important approach for structural topology optimization. By changing a

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difficult topology design problem into a relatively simpler 'sizing' problem, the homogenization technique provides a method for simultaneous shape and topology optimization. As a simplification and variation of the homogenization method, the solid isotropic material with penalization(SIMP) method has many advantages and has been widely considered [2,17,31]. The basic idea of SIMP method is to use of a fictitious isotropic material. The elasticity tensor of this material is assumed to be a penalized material density function which is represented by an exponent parameter. However, both of the homogenization method and the SIMP method have a problem of numerical instability such as checkerboards, mesh-dependencies and local minimal solutions [7,20]. Furthermore, there is an approach called the evolutionary structural optimization (ESO) method for structural topology optimization which was proposed in [30, 33]. In this method, the design domain is discretized by a finite element mesh, and shape and topology of the structure are obtained by progressively removing the material region with low stress.

Recently, the level set method, which was first developed for tracking the propagation of fluid interface [14], has been successfully used to solve a wide range of shape and topology optimization problems [1,27]. Such method implicitly represents the geometric boundary by a higher dimensional function. Generally, a signed distance function is used to represent the level set function and trace the evolution of the boundaries with respect to time by solving Hamilton-Jacobi partial differential equation [8, 15], with a suitable normal velocity which is the moving boundary velocity normal to the interface. During the evolution, the so-called re-initialization [19,21] process should be performed periodically to ensure that the level set function is always a signed distance function. This is a sufficient strategy to maintain numerical accuracy, but it has strong computational difficulties and is quite inefficient [28].

In order to avoid the unnecessary signed distance function and the re-initialization of level set function, an alternative piecewise constant level set (PCLS) method was proposed by Lie-Lysaker-Tai [4,9,10]. The PCLS method was originally used to handle image segmentation [11], see also [25]. In [5,12], the PLCS method was successfully applied to solve inverse problems and interface problems. Wei and Wang [29] also proposed a PCLS method to solve topology optimization problems. Compared with the standard level set method, the PLCS method can easily create small holes without topological derivatives during the evolution. Thus PLCS method reduces dependence on the initial design. However, the PLCS method needs to add a piecewise constant constraint to the level set function.

Binary level seti (BLS) method, as a special PCLS method, was developed in [9, 10] for problems involving the motion of curves and surfaces. Later it was applied to solve elliptic inverse problems with discontinuous coefficients [13] and elliptic shape optimization problems [36], both of them achieved good results. Compared with general PCLS method, the biggest advantage of BLS method is that it only requires N level set functions to represent a structure of $m=2^N$ different material phases. For example, four material phases can be represented using only two level set functions. The level set function in BLS method has only two values 1 and -1.